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THE FORMAL DEMOGRAPHY OF MIGRATION AND REDISTRIBUTION: MEASUREMENT AND DYNAMICS

Andrei Rogers

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Preface

Interest in human settlement systems and policies has been a critical part of urban-related work at IIASA since its inception. Recently this interest has given rise to a concentrated research effort focusing on migration dynamics and settlement patterns. Four sub-tasks form the core of this research effort:

I. the study of spatial population dynamics;

II. the definition and elaboration of a new research area called demometrics and its application to migration analysis and spatial population forecasting;

III. the analysis and design of migration and settlement policy;

IV. a comparative study of national migration and settlement patterns and policies.

This paper, the fourteenth in the dynamics series, is an overview of IIASA's research in the measurement and analysis of migration and population redistribution patterns. It draws on a number of earlier IIASA publications and strives to develop an overall perspective that links the previous research papers.

Related papers in the dynamics series, and other publications of the migration and settlement study, are listed on the back page of this report.

Andrei Rogers
Chairman
Human Settlements
and Services Area

April 1978

Abstract

This paper is a broad overview of migration and redistribution research currently being carried out at IIASA. Fundamental concepts regarding problems of migration measurement are set out and several multiregional demographic models dealing with the redistributional dynamics of national populations are outlined.

Acknowledgements

Any paper reviewing one's past work sharply focuses intellectual debts. This effort is no exception. As will be evident to the reader, I have been greatly influenced by the scholarly contributions of two outstanding mathematical demographers: Ansley Coale and Nathan Keyfitz, and have been generously assisted in my own research by four former graduate students and subsequent colleagues at IIASA: Luis Castro, Jacques Ledent, Richard Raquillet, and Frans Willekens. As the many references to our joint papers indicate, I have borrowed liberally from this collaborative work.

The Formal Demography of Migration and Redistribution: Measurement and Dynamics

Contents

The Formal Demography of Migration and Redistribution: Measurement and Dynamics

The unexpected postwar baby boom in the United States had a salutary influence on demographic research in that it stimulated studies of improved methods for measuring fertility and for understanding the dynamics by which it, together with mortality, determines the age composition of a population. Because attention was principally directed at national population growth, measurement of internal migration and the spatial dynamics by which it affects national patterns of redistribution were neglected. This neglect led Dudley Kirk (1960) to conclude, in his 1960 Presidential Address to the Population Association of America, that the study of migration was the "stepchild" of demography. Sixteen years later, Sidney Goldstein echoed a similar theme in his Presidential address to the same body:

> ...improvement in the quantity and quality of our information on population movement has not kept pace with the increasing significance of movement itself as a component of demographic change...Redistribution has suffered far too long from neglect within the profession.... It behooves us to rectify this situation in this last quarter of the twentieth century, when redistribution in all its facets will undoubtedly constitute a major and increasingly important component of demographic change... (Goldstein, 1976. pp. 19-21).

Improved methods for measuring migration and understanding its important role in spatial population dynamics have been receiving increasing attention in recent years. The search for improved methods for measuring migration has, for example, stimulated research on the construction of multiregional life tables and demographic accounts (Rogers, 1973a,b; Schoen, 1975; Rogers and Ledent, 1976; Rees, 1977; Rees and Wilson, 1977), and the need for a better understanding of spatial population dynamics has fostered mathematical analyses of the fundamental processes of spatial population growth and redistribution (Rogers, 1966, 1968 and 1975a; Stone, 1968; Drewe, 1971; Le Bras, 1971; Feeney, 1970 and 1973; Willekens, 1977).

In this paper I shall describe some of the work that my colleagues and I have carried out during the past decade in the course of searching for more rigorous methods for measuring migration and establishing the fundamental redistributional dynamics through which it influences the evolution of spatial human populations. The first part of the paper deals with measurement, the second with dynamics.

1. MEASUREMENT

The literature on migration has until recently presented a curiously ambivalent position with regard to migration measurement. This ambivalence is particularly striking because of the contrast it poses with respect to the corresponding demographic literature in mortality and fertility (natality)---a literature that is richly endowed with detailed discussions of measurement problems. Haenszel (1967) attributes this paradox to the strong influence of Ravenstein's early contributions to migration analysis:

> Work on migration and population redistribution appears to have been strongly influenced by the early successes of Ravenstein in formulating "laws of migration". Subsequent papers have placed a premium on the development and testing of new hypotheses rather than on descriptions of facts and their collation...This is in contrast to the history of vital statistics. While Graunt more than two centuries before Ravenstein, had made several important generalizations from the study of "bills of mortality" in London, his successors continued to concentrate on descriptions of the forces of mortality and natality by means of rates based on populations at risk. (Haenszel, 1967, p.260).

It is natural to look to the state of mortality and fertility measurement for guidance in developing measures of migration. Like mortality, migration may be described as a process of interstate transfer; however, death can occur but once, whereas migration is potentially a repetitive event. This suggests the adoption of a fertility analog; but the designation of spatial boundaries introduces difficulties in migration measurement that do not arise in fertility analysis.

Migration measurement can usefully apply concepts borrowed from both mortality and fertility analysis, modifying them where necessary to take into account aspects that are peculiar to migration. From mortality analysis, migration can borrow the notion of the life table, extending it to include increments as well as decrements, in order to reflect the mutual interaction of several regional cohorts (Rogers, 1973a,b and 1975a; Rogers and Ledent, 1976). From fertility analysis, migration can borrow well-developed techniques for graduating age-specific schedules (Rogers, Raquillet, and Castro, 1978). Fundamental to both "borrowings" is a workable definition of migration rate.

1.1 Migration Rates

At given moments during the course of a year, or some such fixed interval of time, a number of individuals living in a particular community change their regular place of residence. Let us call such persons mobiles to distinguish them from those individuals who did not change their place of residence, i.e., the nonmobiles. Some of the mobiles will have moved to a new community of residence; others will simply have transferred their household to another residence within the same community. The former may be called movers, the latter are relocators. A few in each category will have died before the end of the unit time interval.

Assessing the situation with respect to the start and the end of the unit time interval, we may divide movers who survived to the end of the interval into two groups: those living in the same community of residence as at the start of the interval and those living elsewhere. The first group of movers will be referred to as surviving returnees, the second will be called surviving migrants. An analogous division may be made of movers who died before the end of the interval to define nonsurviving returnees and nonsurviving migrants.

A move, then, is an event: a separation from a community. A mover is an individual who has made a move at least once during a given interval of time. A migrant (i.e., a surviving or nonsurviving migrant), on the other hand, is an individual who at the end of the given time interval no longer inhabits the same community of residence as at the start of the interval. (The act of separation from one state is linked to an addition to another). Thus paradoxically, a multiple mover may be a nonmigrant by our definition. This is illustrated by life line C in the multiregional Lexis diagram in Figure 1 below. Because this particular mover returned to the initial place of residence before the end of the unit time interval, no "migration" took place.*

The focus on migrants instead of on movers reflects the need at some point to calculate probabilities. As Haenszel (1967) has observed:

> the label "migration"had been applied to two related, but different, universes of discourse - a population of "moves" and a population of "people who move". A universe of "moves" can be generated by simultaneous classification of individuals by initial and subsequent place of residence, and the data provide useful descriptions of population redistribution. Such results, however, do not lend themselves to probability statements. Probabilities can be computed only for denumerable populations at risk, whether they be people, telephone poles, or transistors. (Haenszel, 1967, p.254).

The simplest and most common measure of migration is the crude migration rate, defined as the ratio of the number of migrants, leaving a particular population located in space and time, to the average number of persons (more exactly, the number of person-years) exposed to the risk of becoming migrants.** As in the case of fertility rates: "The base is two-dimensional because events require both actors and the passage of time. Most of the measures discussed...consist of such rates, calculated

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*We define migration to be the transition between states experienced by a migrant.

**Because data on nonsurviving migrants are generally unavailable, the numerator in this ratio often excludes them.

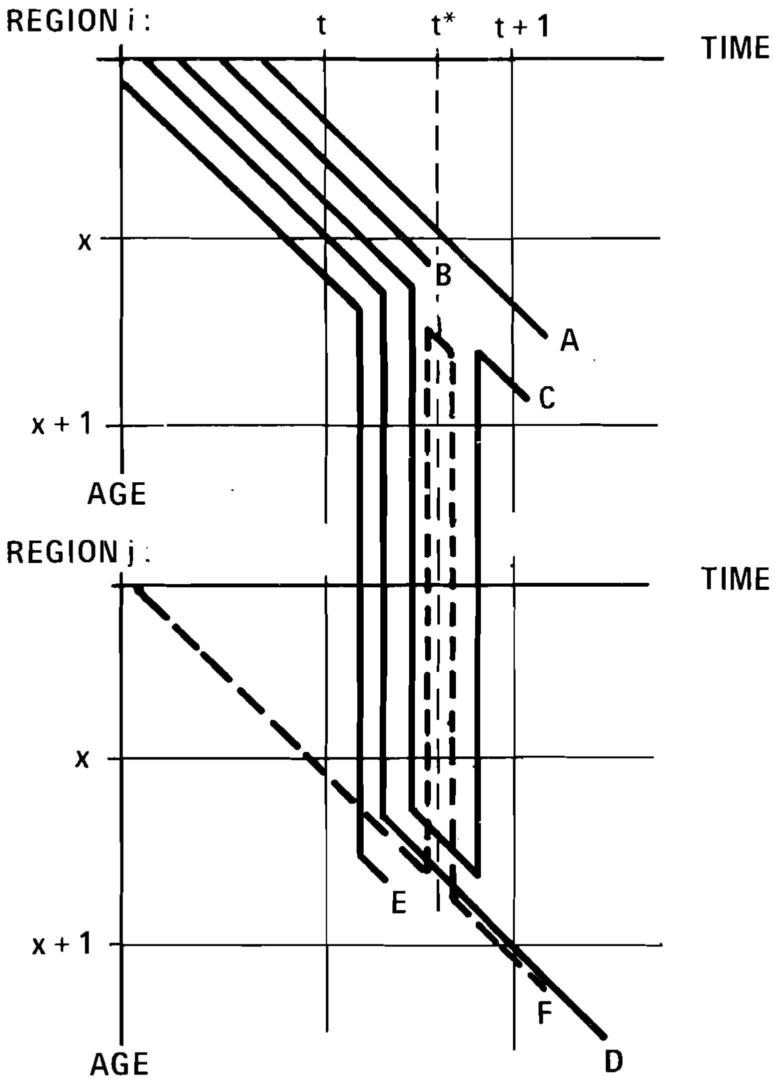

Figure 1. Two-region lexis diagram.

Source: Rogers (1973b).

for ever more refined definitions of the exposure base". (Ryder, 1966, pp.287-288).

Because migration is highly age selective, with a large majority of migrants being the young, our understanding of migration patterns and dynamics is aided by computing migration rates for each age. Weighting each of these rates by the proportion of total population exposure contributed by persons of that age and summing over all ages of life gives the gross migraproduction rate (GMR), the migration analog of the gross reproduction rate.

Although it has been frequently asserted that migration is sex selective, with males being more migratory than females, recent research indicates that sex selectivity in migration is much less pronounced than age selectivity, and that it is less uniform across time and space. Nevertheless, because most models and analyses of population dynamics distinguish between the sexes, most migration measures do also.

Under normal national statistical conditions, point-to-point movements are aggregated into streams between one civil division and another; consequently, the level of interregional migration depends on the size of the areal units selected. Thus, if the areal unit chosen is a minor civil division such as a county or commune, a greater proportion of residential relocation will be included as migration than if the areal unit chosen is a major civil division such as a state or province. Moreover, migration occurs over time as well as across space; therefore studies of its patterns must measure its occurrence with respect to a time interval, as well as over a system of geographical areas. In general, the longer the time interval, the larger will be the number of return movers and, therefore, the more the count of migrants will understate the number of inter-area movers. The impact of these spatial and temporal consolidations may be expressed analytically, and their influence on migration measurement and population dynamics may then be assessed.

A fundamental aspect of migration is its change over time. A time series of age-specific migration rates may be usefully set out in the form of a table with ages for rows and calendar years for columns (i.e., paralleling the format of the Lexis diagram in Figure 1). Such a table yields two sets of summary indices of migration. The column sums give a time series of period gross migraproduction rates. Diagonal sums give cohort gross migraproduction rates (the rates of a cohort of individuals born in the same year). The two series of GMRs normally will differ, with the period series generally fluctuating more than the cohort series.

As Ryder (1964) has shown for the case of fertility, period and cohort reproduction rates will differ whenever the age distribution of childbearing varies from one cohort to another. An analogous result holds for migration. Period gross migraproduction rates become inflated if the mean age of migration declines monotonically from cohort to cohort. Conversely, if declining economic conditions lead potential migrants to delay their migration, current period indices of migration level may decline only to be followed by a compensatory increase in the future.

The usefulness of a cohort approach in migration as in fertility analysis lies in the importance of historical experience to the explanation of current behavior. As Morrison (1970) points out, migration is induced by transitions from one stage of the life cycle to another, and "chronic" migrants may artificially inflate the migration rates of origin areas heavily populated with migration-prone individuals. Both influences on period migration rates are readily assessed by a cohort analysis.

It is the migration of a period, however, and not that of a cohort that determines the sudden redistribution of a national population in response to economic fluctuations, and it is period migration rates that are needed to calculate spatial population projections.

Current period migration indices do not distinguish trend from fluctuation and therefore may be distorted; current cohort migration indices are incomplete. Thus it may be useful to draw on Ryder's (1964) translation technique to change from one to the other. As Keyfitz (1977, p.250) observes, the cohort and period moments in Ryder's formulae can "be interpreted, not as child-bearing, but as mortality, marriage, school attendance, income, or some other attribute of individuals". Migration is clearly such an attribute.

The importance of historical experience in interpreting and understanding current migration behavior led Peter Morrison (1970, p.9) to define the notion of staging as being "any linkage between a prior sequence and subsequent migration behavior". Morrison recognizes four kinds of staging: geographic, life-cycle, socioeconomic, and experiential. Geographical staging refers to return migration and to what is conventionally understood to mean "stage migration", i.e., the idea that migrants tend to move to places not very dissimilar from those they left behind. Life-cycle staging views migrations as arising out of breaks in an individual's or household's life cycle, such as entry into the labor force, marriage, child rearing, retirement. Socioeconomic staging sees migration sequences as being conditioned by sociostructural factors such as occupation, educational attainment, and income level. Finally, experiential staging refers to movement experience in terms of number of previous moves and duration since the last move. It is the "parity" variable of migration analysis.

Calculations of migration rates of increasing specificity seek to unconfound the "true" migration rates from weights that reflect the arithmetical influence of the past. This process of measuring migration "at different levels of specificity of occurrence and exposure yields products which draw ever finer distinctions between current behavior and the residue of past behavior reflected in the exposure distribution at any time" (Ryder, 1975, p.10).

Such products may be weighted and aggregated to produce the "crude" rates of higher levels of aggregation. For example, the age-sex-specific migration rate is a weighted aggregation with respect to the migration "parity-duration" distribution just as the crude migration rate is a weighted aggregation with respect to the age-sex distribution.

1.2 Migration Schedules

The most prominent regularity exhibited by empirical schedules of age-specific migration rates is the selectivity of migration with respect to age. Young adults in their early twenties generally show the highest migration rates and mid-teenagers the lowest. The migration rates of children mirror those of their parents; thus the migration rates of infants exceed those of adolescents. Finally, migration streams directed toward regions with warmer climates and cities with relatively high levels of social services and cultural amenities often exhibit a "retirement peak" at ages in the mid-sixties.

Figure 2 illustrates a typical age-sex-specific migration schedule with a retirement peak. Several important points along the age profile may be identified: the low point, x_l, the high peak, x_p, and the retirement peak, x_r. Associated with the first two points is the labor force shift, X, which is defined to be the difference in years between the ages of the low point and the high peak, i.e., $X = x_p - x_l$. Associated with this shift is the jump, B, the increase in the migration rate of individuals aged x_p over those aged x_l.

The close correspondence between the migration rates of children and those of their parents suggests another important shift in observed migration schedules. If, for each point x on the pre-labor force part of the migration curve, we obtain by interpolation the point, $x + A_x$ say, with the identical rate of migration on the labor force curve, then the average of the values of A_x, calculated for the first dozen or so years of age will be defined to be the observed parental shift, A.

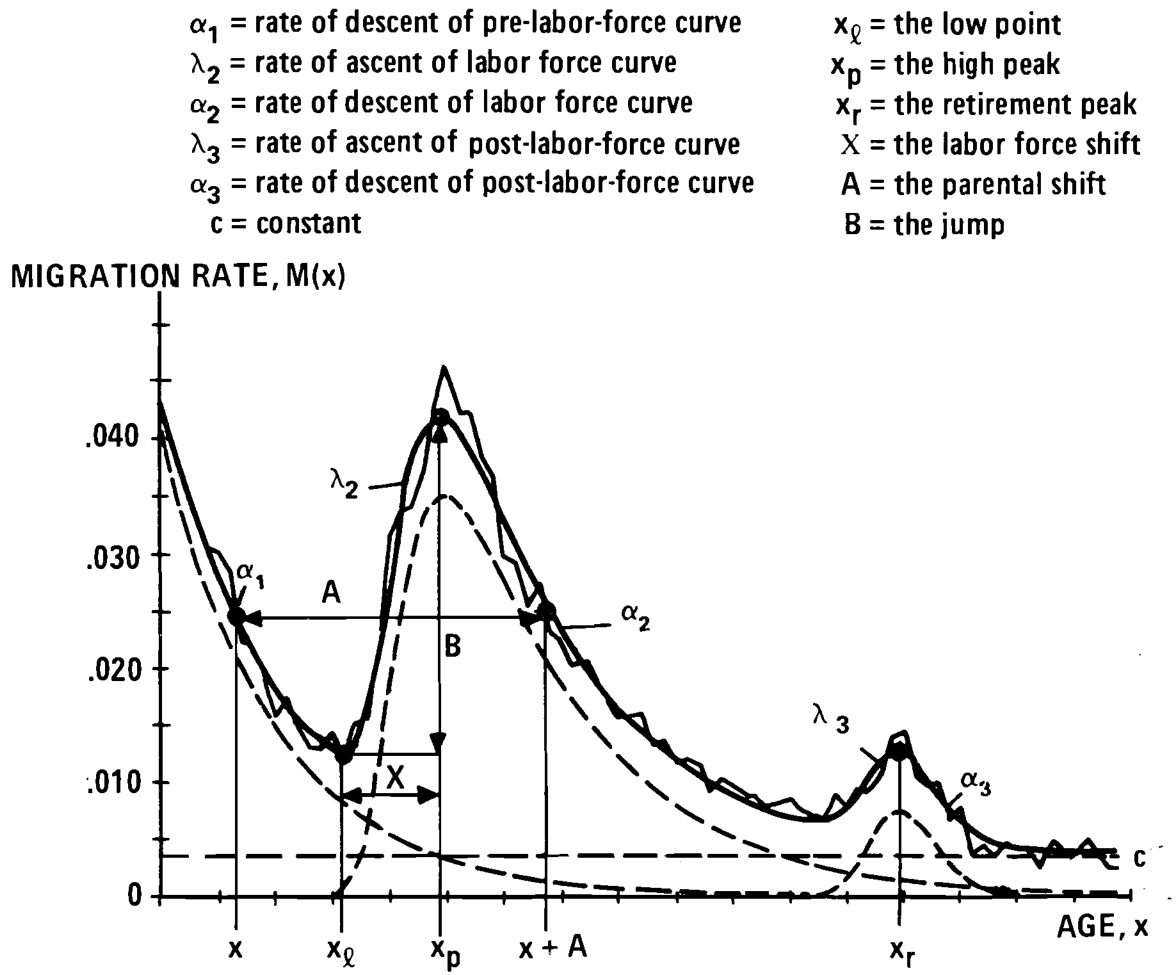

Figure 2. The model migration schedule.

Source: Rogers, Raquillet and Castro (1978).

A particularly useful approach for summarizing and analyzing the regularities present in observed migration schedules is a description of such schedules as the sum of four components:

1) a single negative exponential curve of the pre-labor force ages, with its rate of descent, α_1,

2) a left-skewed unimodal curve of the labor force ages with its rates of ascent and descent, λ_2 and α_2, respectively,

3) an almost bell-shaped curve of the post-labor force ages with its rates of ascent and descent, λ_3 and α_3, respectively, and

4) a constant curve c, the inclusion of which improves the quality of fit provided by the mathematical expression of the schedule.

The decomposition described above suggests the following simple sum of four curves (Rogers, Raquillet, and Castro, 1978):

$$\begin{aligned} M(x) &= a_1 e^{-\alpha_1 x} \\ &+ a_2 e^{-\alpha_2 (x-\mu_2) - e^{-\lambda_2 (x-\mu_2)}} \\ &+ a_3 e^{-\alpha_3 (x-\mu_3) - e^{-\lambda_3 (x-\mu_3)}} \\ &+ c \qquad\qquad , x = 0,1,2,\ldots \end{aligned} \tag{1}$$

The "full" model schedule in (1) has 11 parameters: a_1, α_1, a_2, α_2, μ_2, λ_2, a_3, α_3, μ_3, λ_3, and c. Migration schedules without a retirement peak may be represented by a "reduced" model with 7 parameters, because in such instances the third component of (1) is omitted. The profile of the full model schedule is defined by 7 of the 11 parameters: α_1, α_2, μ_2, λ_2, α_3, μ_3, and λ_3.

Both the labor force and the post-labor force components in (1) are described by the "double exponential" curve formulated by Coale and McNeil (1972) for their studies of nuptiality and fertility. It seems likely that their model can be transformed into one of labor force participation and migration by reinterpreting:

1) entry into the marriage market as entry into the job market,
2) marital search as job search,
3) first marriage frequency as first job frequency, and
4) proportion ever married as proportion ever active.

Migration schedules of the form specified in (1) may be classified into <u>families</u> according to the values taken on by their principal parameters. For example, we may distinguish those schedules with a retirement peak from those without; or we may refer to schedules with relatively low or high values for the rate of ascent λ_2. In many applications, it is also meaningful and convenient to characterize migration schedules in terms of several of the fundamental measures illustrated in Figure 2, such as the low point x_1, the high peak x_p, the labor force shift X, the parental shift A, and the jump B.

In migration schedules without a retirement peak and with a given value of the parental shift, the labor force shift varies approximately as a function of the rate of descent α_2 and the rate of ascent λ_2, (Rogers, Raquillet, and Castro, 1978, Figure 16). Thus, for a given set of values of x_1, x_p, α_2, and A, it is possible to infer the values of λ_2 and μ_2. Given $x_p - x_1$, α_2 and A, we may obtain λ_2. With values for λ_2, α_2, and A, one can obtain the values of $x_p - \mu_2$, and therefore of μ_2. With values for α_2, λ_2, and μ_2 we have the <u>profile</u> (but not the <u>level</u>) of a migration schedule. To obtain the level we also need values for a_1, a_2, and c.

The shape, or <u>profile</u>, of an age-specific schedule of migration rates is a feature that may be usefully studied independently of its intensity, or <u>level</u>. This is because there is considerable empirical evidence that although the latter tends to vary significantly from place to place, the former is remarkably similar in various localities. Some evidence on this point appears below, in Section 1.4 of this paper. We now consider the measurement of migration levels.

The level of migration, like that of mortality, can be measured in terms of an expected duration time, for example, the fraction of a lifetime that is expected to be lived at a particular location. However, like fertility, migration is a potentially repetitive event, and its level therefore can be expressed in terms of an expected number of migrations per person.

The most common demographic measure of level is the notion of expectancy. Demographers often refer to life expectancies, for example, when speaking about mortality, and to reproduction expectancies when discussing fertility. Migration expectancies have been used in migration studies (Wilber, 1963, and Long, 1973). However, their definitions have been nonspatial; migration was viewed as an event occurring in a national population rather than as a flow arising between regional populations.

The study of spatial population dynamics can be considerably enriched by explicitly identifying the locations of events and flows. This permits one to define spatial expectancies such as the expectation of life at birth or the net reproduction rate of individuals born in region i (respectively, ${}_ie(0)$ and ${}_iNRR$, say), and the expected allocation of this lifetime or rate among the various constituent regions of a multiregional population system (${}_ie_j(0)$ and ${}_iNRR_j$, respectively, $j = 1,2,...,m$). For example, it has been estimated (Rogers, 1975a) that the expectation of life at birth of a California-born woman exposed to the 1958 U.S. schedules of mortality and migration would be 73.86 years, out of which 24.90 years would be lived outside of California. The net reproduction rate of such a woman, on 1958 fertility rates, would be 1.69, with 0.50 of that total being born outside of California.

A spatial migration expectancy based on duration times, e.g., the expected number of years lived in region j by individuals born in region i, may be complemented by an alternative definition of spatial migration expectancy--one reflecting a view of migration as a recurrent event. Just as a net reproduction rate can be apportioned among the constituent regions of a multiregional system, so too can a net migraproduction rate, NMR say, be similarly disaggregated by place of birth and place of residence.

The net migraproduction rate ${}_{i}NMR_{j}$ describes the average lifetime number of migrations made out of region j by an individual born in region i. The summation of ${}_{i}NMR_{j}$ over all regions of destination ($j \neq i$) gives ${}_{i}NMR$, the net migraproduction rate of individuals born in region i, i.e., the average number of migrations an i-born person is expected to make during a lifetime.

The gross migraproduction rate measures the intensity of migration between two regions at a particular point in time. The measure, therefore, has a basically cross-sectional character, in contrast to the NMR which measures the intensity of migration over a lifetime. Consequently, the gross migraproduction rate often may prove to be a more useful measure than the net rate in that it is a "purer" indicator of migration, in the same sense as the gross reproduction rate. However, the gross rate measures the intensity of migration at a given moment and not over a lifetime. Hence, in instances where return migration is an important factor, the gross rate and the net rate may give differing indications of geographical mobility.

Table 1 shows that the allocation of the gross migraproduction rate from the Northeast region to the South region in the United States was larger in 1968 than the allocation to the same destination of the West region's gross rate (${}_{1}\varepsilon_{3} = 0.5525 > {}_{4}\varepsilon_{3} = 0.4853$). Yet the opposite was true of the corresponding allocations of the net rate (${}_{1}\gamma_{3} = 0.0965 < {}_{4}\gamma_{3} = 0.1008$). The cause of this reversal was the significantly higher return migration to the West region (${}_{3}\varepsilon_{4} = 0.3302 > {}_{3}\varepsilon_{1} = 0.2606$). Thus, because of the influence of return migration, the lifetime level of geographical mobility to the South region of a baby girl born in the Northeast region was lower, on 1968 rates of migration and mortality, than the corresponding mobility to the same destination of a baby girl born in the West region. The 1968 intensity of geographical mobility to the South region, however, was higher from the Northeast region than from the West region.

Table 1. Gross and net migraproduction rates and allocations by region of residence and region of birth: United States female population, 1968.

Source: Rogers, (1975b), pp.9 and 11.

A. Gross migraproduction rates and allocations: ${}_iGMR_j$ and ${}_i\varepsilon_j$

Region of Birth	Region of Residence				Total
	1	2	3	4	
1. Northeast	-- (--)	0.1258 (0.2137)	0.3253 (0.5534)	0.1377 (0.2339)	0.5889 (1.00)
2. North Central	0.0978 (0.1438)	-- (--)	0.3296 (0.4847)	0.2526 (0.3715)	0.6801 (1.00)
3. South	0.1462 (0.2605)	0.2296 (0.4092)	-- (--)	0.1853 (0.3303)	0.5611 (1.00)
4. West	0.1005 (0.1531)	0.2374 (0.3616)	0.3186 (0.4853)	-- (--)	0.6564 (1.00)

B. Net migraproduction rates and allocations: ${}_iNMR_j$ and ${}_i\gamma_j$

Region of Birth	Region of Residence				Total
	1	2	3	4	
1. Northeast	0.4178 (0.7756)	0.0364 (0.0675)	0.0520 (0.0965)	0.0326 (0.0604)	0.5387 (1.00)
2. North Central	0.0233 (0.0392)	0.4665 (0.7833)	0.0547 (0.0919)	0.0510 (0.0857)	0.5956 (1.00)
3. South	0.0320 (0.0586)	0.0578 (0.1058)	0.4116 (0.7538)	0.0447 (0.0818)	0.5460 (1.00)
4. West	0.0242 (0.0398)	0.0575 (0.0946)	0.0613 (0.1009)	0.4649 (0.7648)	0.6078 (1.00)

1.3 Migration Probabilities

Vital statistics and censuses of the kind regularly collected in most developed nations provide the necessary data for the computation of rates. They may be used to answer questions, such as: what is the current rate at which 40-year old males are dying from heart disease? or at which 30-year old women are bearing their second child? But many of the more interesting questions regarding mortality and fertility patterns are phrased in terms of probabilities, for example: what is the current probability that a man aged 40 will outlive his 38-year old wife, or that she will bear her third child before she is 45?

Demographers normally estimate probabilities from observed rates by developing a life table. Such tables describe the evolution of a hypothetical cohort of babies born at a given moment and exposed to an unchanging age-specific schedule of vital rates. For this cohort of babies, they exhibit a number of probabilities for changes of state, such as dying, and develop the corresponding expectations of years of life spent in different states at various ages.

The simplest life tables recognize only one class of decrement, e.g., death, and their construction is normally initiated by estimating a set of age-specific probabilities of leaving the population, e.g., dying, within each interval of age, $q(x)$ say, from observed data on age-specific exit rates, $M(x)$ say. The conventional calculation that is made for an age interval five years wide is (Rogers, 1975a, p.12).

$$q(x) = \frac{5M(x)}{1 + \frac{5}{2}M(x)} \quad ,$$

or alternatively,

$$p(x) = 1 - q(x) = [1 + \frac{5}{2}M(x)]^{-1}[1 - \frac{5}{2}M(x)] \quad , \qquad (2)$$

where p(x) is the age-specific probability of remaining in the population, e.g., of surviving, between exact ages x to x + 5.

Simple life tables, generalized to recognize several modes of exit from the population are known as multiple-decrement life tables (Keyfitz, 1968, p.333). They have been applied, for example, in studies of mortality by cause of death, of first marriage and death, of labor force participation and death, and of school attendance and death.

A further generalization of the life table concept arises with the recognition of entries as well as exits. Such increment-decrement life tables (Schoen, 1975) allow for multiple movements between several states, for example, transitions between marital statuses and death, (married, divorced, widowed, dead), or between labor force statuses and death (employed, unemployed, retired, dead).

Multiple-radix increment-decrement life tables that recognize several regional populations each with a region-specific schedule of mortality and several destination-specific schedules of internal migration are called multiregional life tables (Rogers, 1973a,b). They represent the most general class of life tables and were originally developed for the study of interregional migration between interacting multiple regional populations. Their construction is initiated by estimating a matrix of age-specific probabilities of surviving and migrating $\underset{\sim}{P}(x)$ from data on age-specific death and migration rates, $\underset{\sim}{M}(x)$. Rogers and Ledent (1976) show that the equation for this estimation may be expressed as the matrix analog of (2):

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{5}{2}\underset{\sim}{M}(x)]^{-1}[\underset{\sim}{I} - \frac{5}{2}\underset{\sim}{M}(x)] \qquad (3)$$

One of the most useful statistics provided by a life table is the average expectation of life beyond age x, e(x) say, calculated by applying the probabilities of survival p(x) to a hypothetical cohort of babies and then observing their average length of life beyond each age.

Expectations of life in a multiregional life table reflect the influences of mortality and migration. Thus in addition to carrying out their traditional function as indicators of mortality levels, they also serve as indicators of levels of internal migration. For example, consider the regional expectations of life at birth that are set out in Table 2 for the U.S. female population in 1968. A baby girl born in the West, and exposed to the multiregional schedule of mortality and migration that prevailed in 1968, could expect to live an average of 75.57 years, out of which total an average of 11.32 years would be lived in the South. Taking the latter as a fraction of the former, we have in 0.1497 a useful indicator of the (lifetime) migration level from the West to the South that is implied by the 1968 multiregional schedule. (Compare these migration levels with those set out earlier in Table 1).

Life tables are normally calculated using observed data on age-specific vital rates. However, in countries without reliable vital registration systems, recourse is often made to inferential estimation methods that rely on <u>model</u> schedules of mortality or fertility. These methods may be extended to multiregional demographic analysis by the introduction of the notion of a <u>model multiregional life table</u> (Rogers, 1975a, pp.146-154).

Model multiregional life tables approximate the regional mortality and migration schedules of a multiregional population, by drawing on the regularities exhibited by the mortality and migration schedules of comparable populations. A collection of such tables may be entered with empirically determined survivorship proportions (disaggregated by region of birth and region of residence) to obtain the particular combination of regional expectations of life at birth (disaggregated by region of birth and region of residence) that best matches the mortality and migration levels implied by these observed proportions (Rogers, 1975a, pp. 172-189).

Age-specific probabilities of migrating, $p_{ij}(x)$, in empirical multiregional life tables mirror the fundamental regularities exhibited by observed migration rates. The migration risks

Table 2. Expectations of life at birth and migration levels by region of residence and region of birth: United States female population, 1968.

Source: Rogers, (1975b), p. 4.

A. Expectations of life at birth: ${}_ie_j(0)$

Region of Birth	Region of Residence				Total
	1	2	3	4	
1. Northeast	54.13	5.08	10.11	5.25	74.56
2. North Central	3.76	52.14	10.48	8.05	74.44
3. South	5.06	7.88	54.53	6.93	74.40
4. West	3.90	7.94	11.32	52.41	75.57

B. Migration levels: ${}_i\theta_j$

Region of Birth	Region of Residence.				Total
	1	2	3	4	
1. Northeast	0.7260	0.0681	0.1356	0.0704	1.00
2. North Central	0.0506	0.7005	0.1408	0.1081	1.00
3. South	0.0680	0.1060	0.7328	0.0931	1.00
4. West	0.0516	0.1051	0.1497	0.6936	1.00

experienced by different age and sex groups of a given population are strongly interrelated, and higher (or lower) than average migration risks among one segment of a particular population normally imply higher (or lower) than average migration risks for other segments of the same population. This association stems in part from the fact that if socioeconomic conditions at a location are good or poor for one group in the population, they are also likely to be good or poor for other groups in the same population. Since migration is widely held to be a response to spatial variations in socioeconomic conditions, these high intercorrelations between age-specific migration risks are not surprising.

A relatively close accounting of the regularities shown by empirically estimated migration probabilities may be obtained with the zero-intercept linear regression model

$$p_{ij}(x) = \beta(x) \; {}_i\theta_j \quad . \tag{4}$$

Estimates of the regression coefficients $\beta(x)$ may be used in the following way. First, starting with a complete set of multiregional migration levels ${}_i\theta_j$ one calculates the matrix of migration probabilities $\underset{\sim}{P}(x)$ for every age, using equation 4. With $\underset{\sim}{P}(x)$ established, one then may compute the usual life table statistics, such as the various region-specific expectations of life at each age. The collective results of all these computations constitute a model multiregional life table.

1.4 Comparative Analysis

A convenient way to examine regularities in empirical migration patterns is first to scale a collection of observed age-specific migration schedules to a GMR of unity and then to fit them with the model schedule defined in Equation 1. This has been done for a subset of migration schedules collected as part of a comparative study of migration and settlement patterns in developed nations (Rogers and Castro, 1978). The schedules are set out in Figures 3 and 4; their parameters appear in Table 3.

The schedules illustrated in Figures 3 and 4 describe migration out of and into the capital region of each of four nations: Sweden, Great Britain, Bulgaria and Japan. The regional delineations are defined in Rogers and Castro (1978). Observed data by five-year age groups (i.e., histograms) were disaggregated into one-year age groups by graduation-interpolation with the model schedule.

Four of the eleven parameters defining the model schedule refer only to migration level: a_1, a_2, a_3, and c. Their values in Table 3 are for a GMR of unity; to obtain corresponding values for other levels of migration, we simply multiply the four numbers shown in the table by the desired level of GMR. For example, the observed GMR for migration out of the Stockholm region in 1974 was 1.45. Multiplying $a_1 = 0.0285$ by 1.45 gives 0.0413, the appropriate value of a_1 with which to generate the migration schedule having a GMR of 1.45.

The remaining seven model schedule parameters in Table 3 refer to migration profile: α_1, α_2, μ_2, λ_2, α_3, μ_3, and λ_3. Their values remain constant for all levels of the GMR. Taken together, they define the age profile of migration from one region to another (e.g., from the Stockholm region to the rest of Sweden). Schedules without a retirement peak yield only the four profile parameters: α_1, α_2, μ_2, and λ_2.

Set out below the model schedule parameters in Table 3 are several "derived" variables --- variables derived either from the original parameters or from the migration curve generated by them. In addition to the mean age of migration, $\bar{n}$, they are:

(i) the measures of labor force and retirement curve asymmetry: $\sigma_2 = \lambda_2/\alpha_2$, and $\sigma_3 = \lambda_3/\alpha_3$, respectively;

(ii) the ages associated with the low point, x_l, the high peak, x_p, and the retirement peak, x_r;

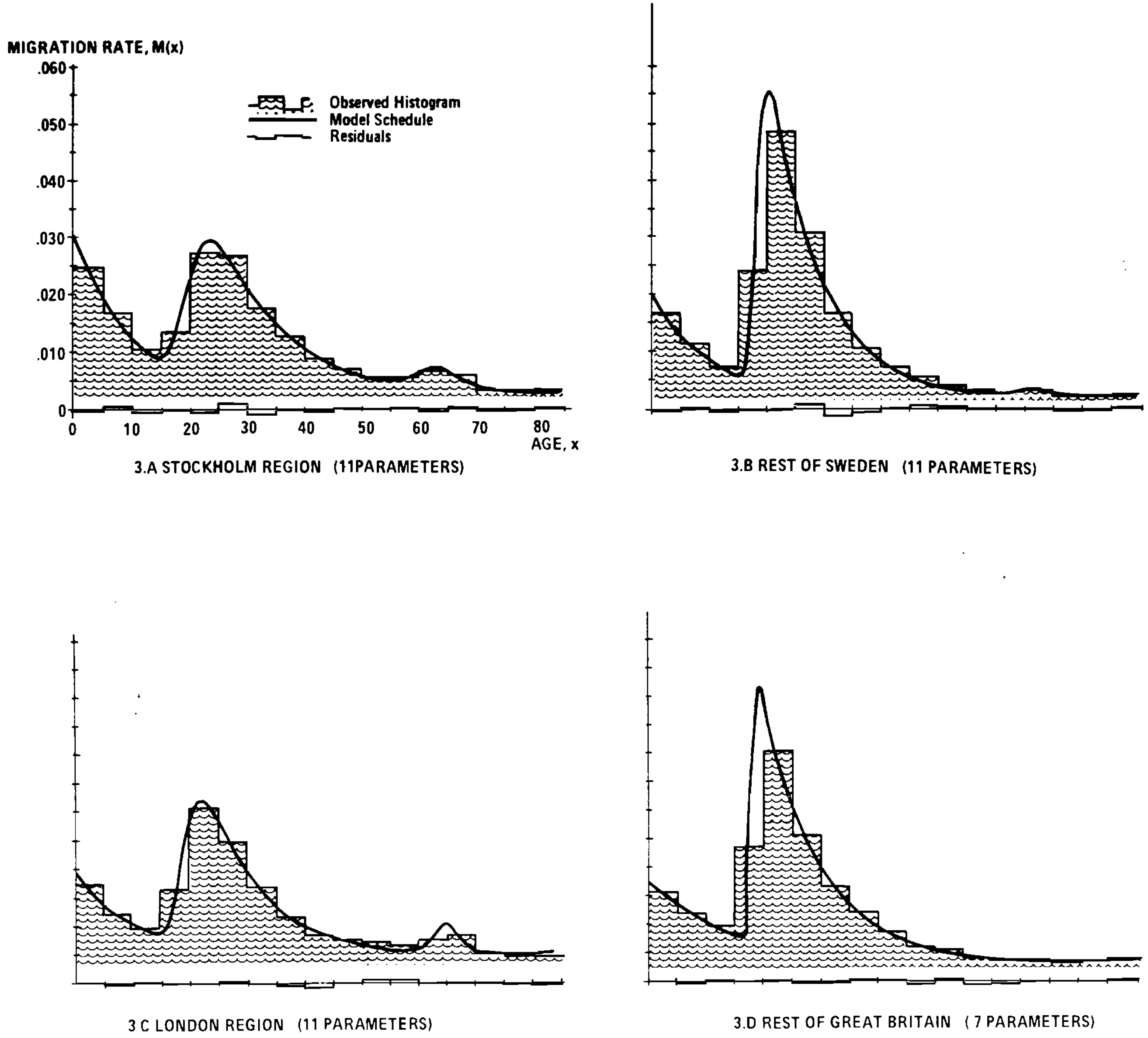

Figure 3. Observed and model migration schedules: Sweden and Great Britain.

Source: Rogers and Castro (1978).

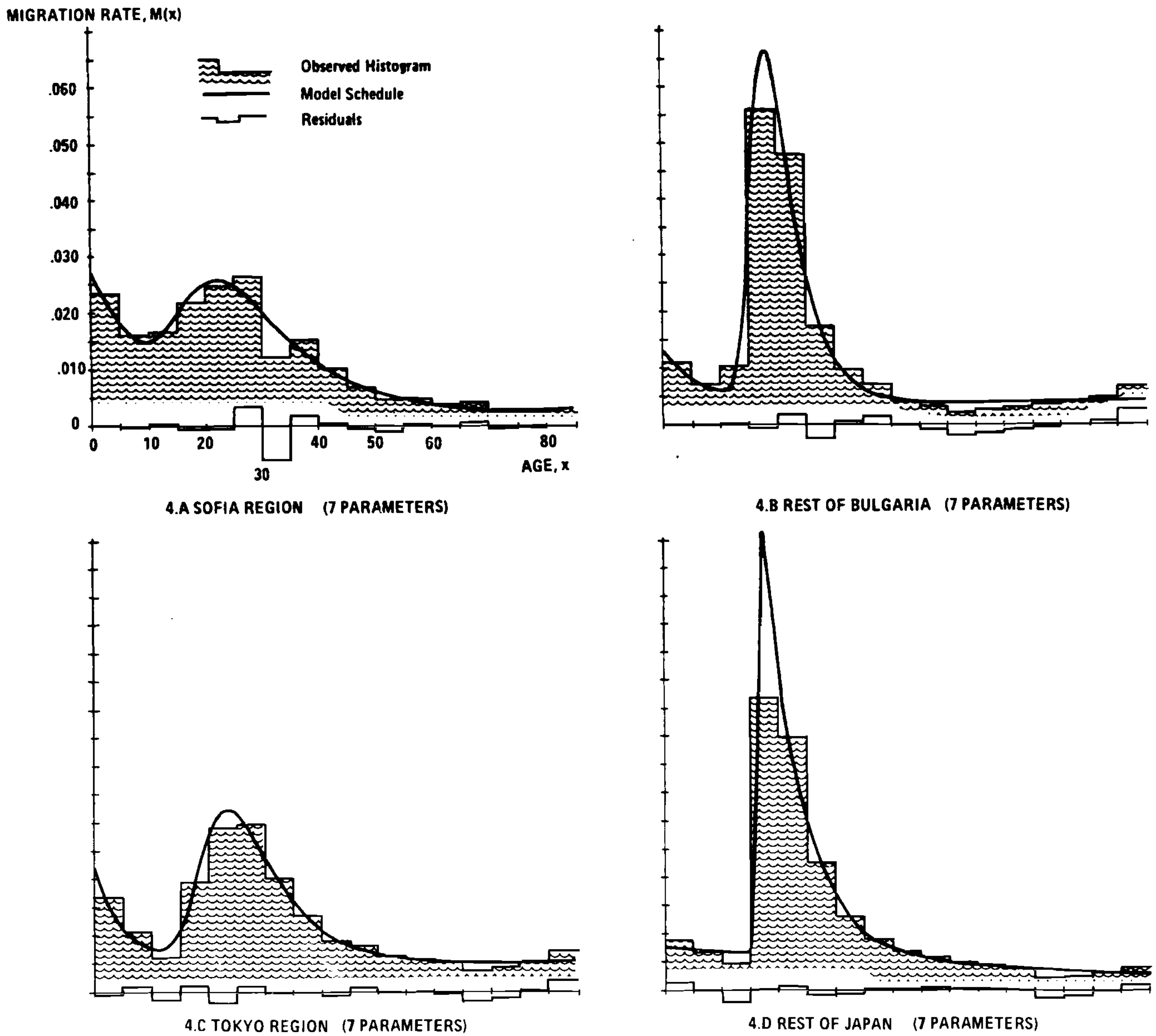

Figure 4. Observed and model migration schedules: Bulgaria and Japan.

Source: Rogers and Castro (1978).

(iii) two shifts: the labor force shift, X, and the parental shift, A; and

(iv) the labor force jump, B.*

Two major classes of migration profiles are illustrated in Figures 3 and 4: migration from the capital region to the rest of the nation, i.e., capital outflow, and migration from the rest of the nation to the capital region, i.e., capital inflow. A cursory visual examination reveals that the two sets of flows exhibit strikingly different age profiles. The parameters and variables in Table 3 articulate more precisely these differences.

The most apparent difference between the age profiles of the capital outflow and inflow migration schedules is the dominance of young labor force migrants in the latter, i.e., proportionately more migrants aged 15 to 24 appear in capital inflow schedules. As a result, the rate of ascent of the labor force curve, λ_2, is always much more steeper in the inflow schedules than in the outflow schedules, i.e., $\lambda_2(i) > \lambda_2(o)$. We shall call this characteristic labor dominance.

A second profile attribute is the degree of asymmetry in the labor force curve of the migration schedule, i.e., the ratio of the rate of ascent λ_2, to the rate of descent α_2, designated by σ_2 in Table 3. In all of the four countries examined, the labor force curve of the capital inflow profile is more asymmetric than that of the corresponding outflow profile, i.e., $\sigma_2(i) > \sigma_2(o)$. We shall refer to this characteristic as labor asymmetry.

Examining the observed rates of descent of the labor and pre-labor force curves, α_2 and α_1, respectively, we find that they are close to being equal in the outflow schedules of London and Sofia (i.e., $\alpha_2 \doteq \alpha_1$), and quite different in the case of Tokyo (i.e., $\alpha_2 < \alpha_1$).

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* A retirement jump could also be defined and studied in an analogous manner.

In all four capital inflow profiles, however, $\alpha_2(i) > \alpha_1(i)$. Profiles with significantly different values for α_2 and α_1, will be said to be irregular.

A number of derived variables such as x_l, x_p, X, A, and B, tend to move together. For example, labor dominant profiles (e.g., capital inflow schedules) exhibit lower values for x_p and X; on the other hand, profiles that are regular (e.g. capital outflow schedules) show higher values for x_p and X, and lower values for x_l, A, and B.

Finally, the schedules for Japan and Sofia show upturns in the migration rates of post-labor force age groups that do not conform to the retirement curve of the model schedule in Equation 1. This may be an indication that a different model schedule is required, e.g., a reverse negative exponential for the retirement ages. However, the relatively uncertain quality of the data for these particular age groups make such a speculation premature.

In conclusion, the empirical migration data of four industrialized nations suggest the following hypothesis. The migration profile of a typical capital inflow schedule is, in general, more labor dominant, more labor asymmetric, and more irregular than the migration profile of the corresponding capital outflow schedule, and it is much less likely to exhibit a retirement peak.

Table 3. Parameters and variables defining the model migration schedule: Sweden, Great Britain, Bulgaria, and Japan.

Source: Rogers and Castro (1978).

Regions / Parameters and Variables	Sweden 1974		Great Britain 1970		Bulgaria 1975		Japan 1969	
	Stockholm	R.S.	London	R.G.B.	Sofia	R.B.	Tokyo	R.J.
Population (000)	1,487	6,670	17,316	36,871	1,070	7,657	29,496	75,169
GMR	1.45	0.28	1.04	0.44	0.29	0.10	2.60	0.71
a_1	0.0285	0.0189	0.0153	0.0143	0.0257	0.0099	0.0188	0.0079
α_1	0.1032	0.1033	0.1008	0.0687	0.0918	0.1503	0.1986	0.0110
a_2	0.0452	0.0762	0.0446	0.0519	0.0504	0.1549	0.0688	0.0909
α_2	0.0912	0.1151	0.1045	0.1042	0.0901	0.2279	0.1320	0.1528
μ_2	20.16	18.22	19.03	18.26	20.18	17.35	21.69	16.61
λ_2	0.3441	0.8913	0.4585	3.1953	0.1434	0.3735	0.2016	3.3391
a_3	0.0000	0.0000	0.0001					
α_3	0.6851	1.1593	1.2231					
μ_3	79.00	74.81	72.93					
λ_3	0.1148	0.2023	0.2209					
c	0.0029	0.0022	0.0051	0.0035	0.0026	0.0040	0.0051	-0.0002
$\bar{n}$	29.21	27.19	32.90	29.44	27.48	27.46	32.34	28.57
σ_2	3.77	7.74	4.39	30.67	1.59	1.64	1.53	21.85
σ_3	0.17	0.17	0.18					
x_1	15.32	15.97	15.01	17.59	12.00	12.00	12.18	15.90
x_p	23.71	20.48	22.12	19.31	22.33	18.66	23.74	18.00
x_r	63.20	60.15	65.2					
X	8.39	4.51	7.11	1.73	10.32	6.66	11.56	2.10
A	26.72	29.95	29.48	29.36	26.26	27.46	34.49	33.01
B	0.0206	0.0500	0.0235	0.0444	0.0113	0.0610	0.0253	0.0752

2. DYNAMICS

Until about a decade ago, the contribution of internal migration to population growth was assessed in nonspatial terms. The evolution of regional populations affected by migration was examined by adding the contribution of net migration to that of natural increase. The dynamics of redistribution, therefore, were expressed over time but not over space; the evolution of a system of interacting regional populations was studied one region at a time.

Beginning in the mid-1960's, efforts to express the dynamics of spatial change in matrix form began to appear in the demographic literature and had considerable success in describing processes of geographical redistribution in multiregional population systems (Rogers, 1966, 1968, 1975a). Such studies, typically, have focused on a process of change in which a population disaggregated into several classes and set out as a vector, is premultiplied by a matrix that advances the population forward over time, and geographically across space.

The spatial distribution of a multiregional population across its constituent regions and the age compositions of its regional populations are determined by the interactions of fertility, mortality, and interregional migration. People are born, age with the passage of time, reproduce, migrate, and ultimately die. In connecting these events and flows to determine the growth rate of each population, one also obtains the number of people in each region and their age composition.

Spatial processes of population growth and redistribution may be studied with the aid of multiregional generalizations of the discrete Leslie model (Rogers, 1966) or of the continuous Lotka renewal equation (Le Bras, 1971). These formal representations of multiregional population growth and change permit one to examine, for example, the spatial consequences of alternative paths to zero population growth (Rogers and Willekens, 1976 and 1978) and to focus on the mathematical analysis and design of particular intervention policies for redirecting the spatial population system's growth path toward a target multiregional distribution (Rogers, 1968; Willekens, 1976; Willekens and Rogers, 1977). Finally, such models also permit one to examine more rigorously the dynamics of urbanization (Rogers, 1977).

2.1 Population Redistribution

Multiregional generalizations of the classical models of mathematical demography project the numerical consequences, to an initial (single-sex) multiregional population, of a particular set of assumptions regarding future fertility, mortality, and internal migration. The mechanics of such projections typically revolve around three basic steps. The first ascertains the starting age-region distributions and the age-specific regional schedules of fertility, mortality, and migration to which the multiregional population has been subject during a past period; the second adopts a set of assumptions regarding the future behavior of such schedules; and the third derives the consequences of applying these schedules to the initial population.

The discrete model of multiregional demographic growth expresses the population projection process by means of a matrix operation in which a multiregional population, set out as a vector, is multiplied by a growth matrix that survives that population forward through time. The projection calculates the region and age-specific survivors of a multiregional population of a given sex and adds to this total the new births that survive to the end of the unit time interval. This process may be described by the matrix model:

$$\{\underset{\sim}{K}(t+1)\} = \underset{\sim}{G}\{\underset{\sim}{K}(t)\} \quad , \tag{5}$$

where the vector $\{\underset{\sim}{K}(t)\}$ sets out the multiregional population disaggregated by age and region, and the matrix $\underset{\sim}{G}$ is composed of zeroes and elements that represent the various age-region-specific components of population change.

As in the single-region model, survival of individuals from one moment in time to another, say 5 years later, is calculated by diminishing each regional population to take into account the decrement due to mortality. In the multiregional model, however, we also need to include the decrement due to outmigration and the increment contributed by inmigration. An analogous problem is presented by surviving children born during the 5 year interval. Some of these migrate with their parents; others are born after their parents have migrated but before the unit time interval has elapsed.

It is well known that a population that is undisturbed by migration will, if subjected to an unchanging regime of mortality and fertility, ultimately achieve a stable constant age distribution that increases at a constant stable growth ratio, λ say. In Rogers (1966) it is shown that this same property obtains region-by-region in the case of a multiregional population system that is closed to external migration and subjected to an unchanging multiregional schedule of mortality, fertility, and internal migration. Knowledge of the asymptotic properties of such a population projection helps us understand the meaning of observed age-specific birth, death and migration rates. In particular, the quantity $r = 0.2 \ln \lambda$ gives the intrinsic rate of growth that is implied by the indefinite continuation of observed schedules of mortality, fertility, and migration.

A related but equally useful demographic measure is the <u>stable equivalent</u> Y (Keyfitz, 1969) of each region and its proportional allocation across age groups in that region, $C_i(x)$, which is the region's <u>stable age composition</u>. The former may be obtained by projecting the observed multiregional population forward until it becomes stable and dividing the resulting age-region-specific totals by the stable growth ratio λ raised to the n^{th} power, where n is the number of iterations that were needed to achieve stability. Summing across all age groups in a region gives the regional stable equivalent Y_i; dividing the number in each age group in region i by Y_i gives $C_i(x)$, region i's age composition at stability. Finally, dividing each region's stable equivalent by the sum total of all regional stable equivalents gives SHA_i, region i's stable regional share of the total multiregional population at stability.

The growth, spatial distribution, and regional age compositions of a "closed" multiregional population are completely determined by the recent history of fertility, mortality, and internal migration it has been subject to. Its current crude regional birth, death, migration and growth rates are all governed by the interaction of the prevailing regime of growth with the current regional age compositions and regional shares of the total population. The dynamics of such growth and change are clearly illustrated, for example, by the four-region population system exhibited in Tables 4 and 5, and Figure 5, which describe the evolution of the U.S. total population resident in the four Census Regions that collectively exhaust the national territory: 1) the Northeast Region, 2) the North Central Region, 3) the South Region, and 4) the West Region.

The prevailing growth regime is held constant and two sets of spatial population projections are obtained. These offer interesting insights into the growth rates, regional shares, and regional age compositions that evolve from a projection of current trends into the future, taking 1958 and 1968 as alternative base years from which to initiate the projections.

Table 4 shows that between the two base years (1958 and 1968) the regional growth rates of the South and West Regions were higher than the national average, whereas those of the Northeast and North Central Regions were lower. By virtue of the assumption of a linear model and a constant regime of growth, all four regional growth rates ultimately converge to the same intrinsic rate of increase: 0.021810 in the case of the 1958 growth regime, and 0.005699 in the case of the 1968 growth regime. However, what is interesting is that the trajectories converging toward these two intrinsic rates are quite different. Only in the case of the West Region is a decline in the long-run growth rate projected under either of the two observed growth regimes. Also of interest

Table 4. Projected annual regional rates of growth $[r_i(t)]$: total **United** States population.

Source: Rogers and Castro (1976), p.59.

A. Base Year: 1958

Time t \ Region i	1. Northeast	2. North Central	3. South	4. West	Total
1958	0.008484	0.011421	0.016831	0.027227	0.014777
1968	0.009335	0.013217	0.017296	0.026612	0.015896
1978	0.012085	0.015817	0.018111	0.026624	0.017776
1988	0.014067	0.017446	0.019041	0.026256	0.019060
1998	0.016221	0.019284	0.020158	0.026261	0.020483
2008	0.018264	0.020653	0.021190	0.025739	0.021574
Stability			0.021810		

B. Base Year: 1968

Time t \ Region i	1. Northeast	2. North Central	3. South	4. West	Total
1968	0.003808	0.006633	0.011606	0.014698	0.008890
1978	0.005500	0.008549	0.011317	0.014101	0.009734
1988	0.004323	0.006853	0.008900	0.011126	0.007756
1998	0.004663	0.007056	0.008621	0.010408	0.007703
2008	0.005085	0.006953	0.008088	0.009466	0.007435
2018	0.004555	0.006175	0.007204	0.008380	0.006630
Stability			0.005769		

Table 5. Observed and projected regional shares $[SHA_i(t)]$: total United States population.

Source: Rogers and Castro (1976), p.60.

A. Base Year: 1958

Time t \ Region i	1. Northeast	2. North Central	3. South	4. West.	Total
1958	0.2503	0.2955	0.3061	0.1481	1.0000
1968	0.2347	0.2861	0.3122	0.1670	1.0000
1978	0.2202	0.2792	0.3157	0.1850	1.0000
1988	0.2084	0.2740	0.3164	0.2012	1.0000
1998	0.1986	0.2699	0.3161	0.2154	1.0000
2008	0.1907	0.2668	0.3150	0.2275	1.0000
Stability	0.1443	0.2525	0.3061	0.2971	1.0000

B. Base Year: 1968

Time t \ Region i	1. Northeast	2. North Central	3. South	4. West	Total
1968	0.2413	0.2784	0.3090	0.1713	1.0000
1978	0.2306	0.2728	0.3198	0.1768	1.0000
1988	0.2216	0.2699	0.3243	0.1841	1.0000
1998	0.2143	0.2676	0.3280	0.1901	1.0000
2008	0.2082	0.2660	0.3307	0.1950	1.0000
2018	0.2035	0.2647	0.3328	0.1989	1.0000
Stability	0.1764	0.2617	0.3425	0.2194	1.0000

is the substantial difference between the two intrinsic growth rates themselves, which clearly documents the dramatic drop in fertility levels that occurred during the decade in question.

Both in 1958 and in 1968 approximately 31 percent of the U.S. population resided in the South. This regional share remains relatively unchanged in the projection under the 1958 growth regime but increases to over 34 percent under the 1968 growth regime. Thus the ultimate spatial allocation of the national population changed in favor of the South during the decade between 1958 and 1968. According to Table 5, a large part of this change occurred at the expense of the West's regional share, which declined from roughly 30 percent to about 22 percent. Despite this decline, the West's projected share of the national population nonetheless shows a substantial increase over the base year allocation. This increase and that of the South match the decrease in the regional shares of the Northeast and North Central Regions. Thus, under either projection, the "North's" share of the U.S. population is headed for a decline while that of the "South West" is due to increase.

Figure 5 vividly illustrates the impact that a high growth rate has on age composition. The four regional graphs depict both the age compositions observed at the time of the base year and those projected 50 years forward on the assumption of an unchanging regime of growth. Since the regional growth regimes in 1958 produced a relatively high time series of growth rates after a period of 50 years, the age compositions of the left-hand side of Figure 5 show a relatively steep slope. Because the 1968 growth regimes, on the other hand, produced relatively low regional growth rates after 50 years, the regional age compostions on the right-hand side show a relatively shallow slope.

Although the discrete model in (5) is the model normally used for carrying out multiregional population projections, other mathematical analyses of population redistribution often are more readily expressed and studied with the aid of the <u>continuous</u> model of demographic growth.

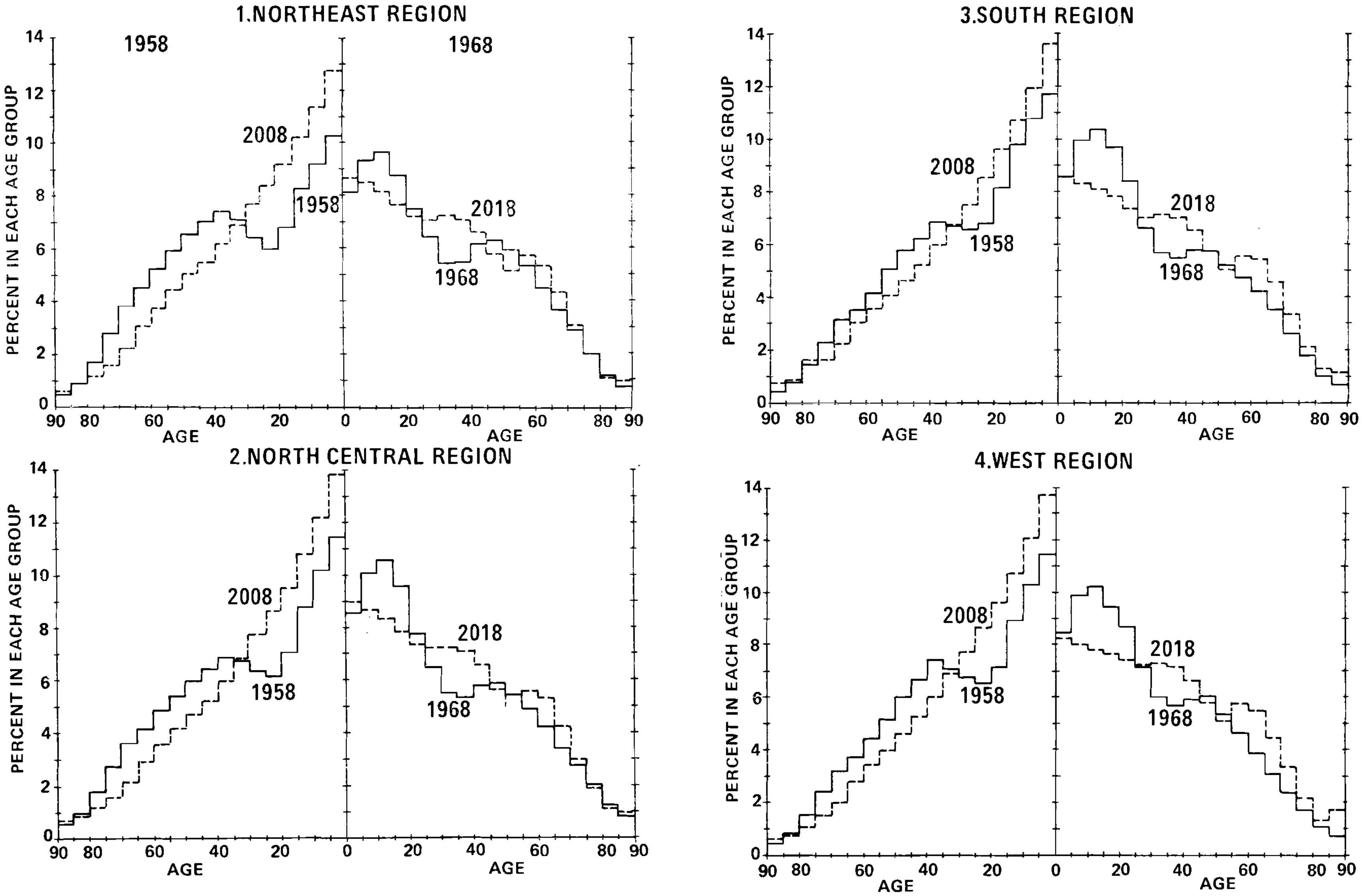

Figure 5. Observed and projected regional age compositions: total United States population.

Source: Rogers and Castro (1976), p.13.

The principal contribution of the continuous model of demographic growth lies in its ability to trace through the ultimate consequences of applying a fixed schedule of age-specific rates of fertility, mortality, and migration to a population of a single sex. By associating the births of a current generation with those of a preceding generation, it develops several important constants that describe the ultimate growth and regional age distributions of such a population.

A continuous model of single-sex population growth may be defined for a multiregional population system by means of a straightforward generalization of the corresponding single-region model. Beginning with the number of female births at time t in each region, $B_i(t)$ say, we note that women aged a to a + da in region i at time t are survivors of those born a years ago and now living in region i at age a, that is $\sum_{j=1}^{m} B_j(t - a)\,{}_j\ell_i(a)da$, where $a \leq t$. At time t, these women give birth to $[\sum_{j=1}^{m} B_j(t - a)\,{}_j\ell_i(a)]m_i(a)da$ children in region i per year. Here ${}_j\ell_i(a)$ denotes the probability that a baby girl born in region j will survive to age a in region i, and $m_i(a)da$ is the annual rate of female childbearing among women aged a to a + da in region i.

Integrating the above expression over all a and focusing on the population at times beyond the last age of childbearing β, gives the homogeneous equation system

$$\{\underset{\sim}{B}(t)\} = \int_0^{\beta} \underset{\sim}{m}(a)\underset{\sim}{\ell}(a)\{\underset{\sim}{B}(t - a)\}da \quad . \tag{6}$$

The matrix product $\underset{\sim}{m}(a)\underset{\sim}{\ell}(a)$, denoted by the matrix $\underset{\sim}{\Phi}(a)$, is the <u>multiregional net maternity function</u>, with which we may associate the moment matrices $\underset{\sim}{R}(n) = \int_0^{\beta} a^n \underset{\sim}{\Phi}(a)da$. To solve (6) we adopt the trial solution $\{\underset{\sim}{B}(t)\} = \{\underset{\sim}{Q}\}e^{rt}$ which when substituted into (6) gives

$$\{\underset{\sim}{Q}\} = [\int_0^{\beta} e^{-ra}\underset{\sim}{\Phi}(a)da]\ \{\underset{\sim}{Q}\} = \underset{\sim}{\Psi}(r)\{\underset{\sim}{Q}\}\ , \qquad (7)$$

where $\underset{\sim}{\Psi}(r)$ is the <u>multiregional characteristic matrix</u>.

We now have reduced our problem from one of solving the integral equation in (6) to that of solving (7) which, unlike (6), is a function of only a single variable, r. To solve for r in (7), we observe that $\{\underset{\sim}{Q}\}$ is the characteristic vector that corresponds to the characteristic root of unity of the matrix $\underset{\sim}{\Psi}(r)$, and r is the number for which that matrix has a characteristic root of unity.

The growth dynamics of empirical populations are often obscured by the influences that particular initial conditions have on future population size and composition. Moreover, the vast quantities of data and parameters that go into a description of such empirical dynamics make it somewhat difficult to maintain a focus on the broad general outlines of the underlying demographic process, and instead often encourage a consideration of its more peculiar details. Finally, studies of empirical growth dynamics are constrained in scope to population dynamics that have been experienced and recorded; they cannot be extended readily to studies of population dynamics that have been experienced but not recorded or that have not been experienced at all. In consequence, demographers frequently have resorted to examinations of the dynamics exhibited by hypothetical <u>model</u> populations that have been exposed to hypothetical <u>model</u> schedules of growth and change.

The study of population dynamics by means of model schedules and model stable populations has been pioneered by Ansley Coale. In a series of articles and books published during the past decade, he and his collaborators have established a paradigm that has become the standard approach of most mathematical demographers. This paradigm is developed in an early study in which Coale and Demeny (1966) present two sets of model (single-region) stable populations that evolve after a long and continued exposure to particular combinations of unchanging schedules of growth. Each population is identified by two non-redundant indices of variation relating to fertility and mortality, respectively, and evolves out of a particular combination of a model life table and an intrinsic rate of growth or gross reproduction rate. The former are referred to as the "growth rate" stable populations; the latter are called the "GRR" stable populations and rely on a model fertility schedule with a given mean age of childbearing $\bar{m}$, which is assumed to be 29 years. Symbolically, the two sets of model stable populations may be expressed as:

1. Growth rate stable popualtions: $f(e(0),r)$;

2. GRR stable populations: $g(e(0),GRR)$,

where $e(0)$ is the expectation of life at birth, r is the intrinsic annual rate of growth, and GRR is the gross reproduction rate.

The paradigm introduced by Coale and Demeny may be extended to multiregional populations. In such an extension, a particular model multiregional life table is linked with an intrinsic rate of growth or set of gross reproduction rates. In the former case one must also specify a set of additional indices that relate to spatial distribution, for example, the spatial distribution of births or of people (Rogers, 1975a, and Rogers and Willekens, 1976). Symbolically, the two sets of model multiregional stable populations may be expressed as:

1. Growth rate multiregional stable populations:
   $f(\underset{\sim}{EXP},r,\underset{\sim}{SRR},\underset{\sim}{\theta})$ or $h(\underset{\sim}{EXP},r,\underset{\sim}{SHA},\underset{\sim}{\theta})$;

2. GRR multiregional stable populations: $g(\underset{\sim}{EXP},\underset{\sim}{GRR},\underset{\sim}{\theta})$,

where $\underset{\sim}{EXP}$ is a diagonal matrix of regional expectations of life at birth, ${}_ie(0)$; $\underset{\sim}{SRR}$ is a matrix of stable radix ratios $\underset{\sim}{SRR}_{ji}$; $\underset{\sim}{SHA}$ is a diagonal matrix of stable regional shares SHA_i; $\underset{\sim}{\theta}$ is a matrix of migration levels ${}_j\theta_i$; and $\underset{\sim}{GRR}$ is a diagonal matrix of regional gross reproduction rates GRR_i. (Alternatively, we could instead have adopted gross migraproduction rates GMR_{ji} in place of the migration levels ${}_j\theta_i$. In this event the matrix $\underset{\sim}{\theta}$ would be replaced by the matrix $\underset{\sim}{GMR}$.)

Tables 6 and 7 set out several specimen model multiregional stable populations that were generated by means of specific combinations of model schedules of fertility, mortality, and migration. The model fertility schedules were obtained by applying Coale and Demeny's (1966) basic age profile, for a mean age of childbearing of 29 years, to different values of GRR; model mortality schedules were taken from their "WEST" family; and the model migration schedules were calculated using the "AVERAGE" regression equations set out in Appendix Table D.2. of Rogers and Castro (1976). Each of the populations in the two tables may be expressed symbolically by any one of the three forms described earlier.

Model multiregional stable populations readily reveal the long-run consequences of particular changes in fertility, mortality, and migration levels. For example, consider several of the more interesting aspects of population dynamics that are manifested in the stable populations presented in Tables 6 and 7. First, identical schedules of regional fertility and mortality produce identical stable regional age compositions. The stable regional shares of such populations, however, will vary inversely with the ratio of their respective migration levels. Second, higher values of the intrinsic growth rate lead to stable (regional) populations that taper more rapidly with age and, in consequence, include a higher proportion of the population below every age. Third, fertility affects not only the rate of growth of a stable population, but also its regional distribution. Fourth, mortality and migration schedules affect the form of the stable regional age compositions and the stable regional shares in an obvious way, and any idiosyncracies in the age patterns of such schedules will be reflected in the age patterns of the corresponding regional populations.

Somewhat surprising is the relative insensitivity of regional age compositions and birth rates to changes in migration levels. For example, consider the case of unequal migration levels with $GRR_1 = 1$, $GRR_2 = 3$, and that with $GRR_1 = 3$, $GRR_2 = 1$. In the first case the region with the larger (by a factor of 2)

Table 6. Model growth rate multiregional (two-region) female stable populations with equal mortality levels: ${}_1e(0) = {}_2e(0) = 70$ years.

Source: Rogers and Castro (1976), p.49.

		Intrinsic Rate of Growth (r)											
		r = 0.00			r = 0.01			r = 0.02			r = 0.03		
		Region			Region			Region			Region		
Growth Rate Set*		1 + 2	1	2	1 + 2	1	2	1 + 2	1	2	1 + 2	1	2
A.	SHA	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000	1.0000	0.5000	0.5000
${}_1\theta_2 = {}_2\theta_1 = 0.3$	b	0.0143	0.0143	0.0143	0.0203	0.0203	0.0203	0.0276	0.0276	0.0276	0.0358	0.0358	0.0358
$SRR_{12} = SRR_{21} = 1$	Δ	0.0143	0.0143	0.0143	0.0103	0.0103	0.0103	0.0076	0.0076	0.0076	0.0058	0.0058	0.0058
	a	37.92	37.92	37.92	32.82	32.82	32.82	28.16	28.16	28.16	24.11	24.11	24.11
B.	SHA	1.0000	0.5999	0.4001	1.0000	0.5919	0.4081	1.0000	0.5839	0.4162	1.0000	0.5762	0.4238
${}_1\theta_2 = 0.2;\ {}_2\theta_1 = 0.4$	b	0.0143	0.0119	0.0179	0.0203	0.0172	0.0249	0.0276	0.0236	0.0331	0.0358	0.0311	0.0422
$SRR_{12} = SRR_{21} = 1$	Δ	0.0143	0.0119	0.0179	0.0103	0.0072	0.0149	0.0076	0.0036	0.0131	0.0058	0.0011	0.0122
	a	37.92	39.24	35.94	32.82	34.20	30.82	28.16	29.52	26.26	24.11	25.38	22.37

*Parameters under stability: regional share, SHA; birth rate, b; absence rate, Δ; average age, a; stable radix ratio, SRR.

Table 7. Model GRR multiregional (two-region) female stable populations with equal mortality levels: ${}_1e(0) = {}_2e(0) = 70$ years.

Source: Rogers and Castro (1976), p.50.

		Gross Reproduction Rate (GRR)								
		$GRR_1 = 1$, $GRR_2 = 1$ Region			$GRR_1 = 2$, $GRR_2 = 1$ Region			$GRR_1 = 3$, $GRR_2 = 1$ Region		
GRR Set*		1 + 2	1	2	1 + 2	1	2	1 + 2	1	2
A.	SHA	1.0000	0.5000	0.5000	1.0000	0.6168	0.3832	1.0000	0.6801	0.3199
	b	0.0131	0.0131	0.0131	0.0232	0.0282	0.0152	0.0331	0.0409	0.0165
${}_1\theta_2 = {}_2\theta_1 = 0.3$	Δ	0.0153	0.0153	0.0153	0.0091	0.0140	0.0010	0.0063	0.0141	-0.0103
	r	-0.0022	——	——	0.0142	——	——	0.0268	——	——
	a	39.08	39.08	39.08	30.80	28.84	33.96	25.34	23.06	30.17
	SRR_{21}	1.000	——	——	0.335	——	——	0.189	——	——
B.	SHA	1.0000	0.6667	0.3333	1.0000	0.7556	0.2444	1.0000	0.7976	0.2024
	b	0.0131	0.0131	0.0131	0.0254	0.0286	0.0156	0.0363	0.0413	0.0167
${}_1\theta_2 = 0.2;\ {}_2\theta_1 = 0.4$	Δ	0.0153	0.0153	0.0153	0.0082	0.0114	-0.0016	0.0057	0.0107	-0.0139
	r	-0.0022	——	——	0.0172	——	——	0.0306	——	——
	a	39.08	39.08	39.08	29.42	28.25	33.04	23.88	22.56	29.09
	SRR_{21}	0.500	——	——	0.176	——	——	0.103	——	——
C.		$GRR_1 = 1$, $GRR_2 = 1$			$GRR_1 = 1$, $GRR_2 = 2$			$GRR_1 = 1$, $GRR_2 = 3$		
${}_1\theta_2 = 0.2;\ {}_2\theta_1 = 0.4$	SHA	1.0000	0.6667	0.3333	1.0000	0.5391	0.4609	1.0000	0.4550	0.5450
	b	0.0131	0.0131	0.0131	0.0208	0.0148	0.0277	0.0293	0.0161	0.0404
	Δ	0.0153	0.0153	0.0153	0.0101	0.0042	0.0171	0.0071	-0.0061	0.0182
	r	-0.0022	——	——	0.0106	——	——	0.0222	——	——
	a	39.08	39.08	39.08	32.52	35.08	29.52	27.22	31.52	23.63
	SRR_{21}	0.500	——	——	1.603	——	——	3.010	——	——

*Parameters under stability: Regional share, SHA; birth rate, b; Absence rate, Δ; average age, a; stable radix ratio, SRR.

outmigration has the higher fertility level; in the second case the situation is reversed. Yet in both instances the population of the region with the higher fertility level has an average age of approximately 23 years and a birth rate of approximately 41 per 1000. This insensitivity to migration behavior does not extend to aggregate systemwide measures, however. For the same example, the intrinsic growth rate and systemwide birth rate are considerably lower in the first case than in the second; the higher fertility region, however, assumes a stable regional share of only 54 percent in the first case but of 80 percent in the second.

Finally, it is important to underscore the powerful influence that past patterns of fertility, mortality, and migration play in the determination of present regional age compositions and shares, inasmuch as the latter arise out of a history of regional births, deaths, and internal migration. For example, a region experiencing high levels of fertility will have a relatively younger population, but if this region also is the origin of high levels of outmigration, a large proportion of its young adults will move to other regions, producing a higher growth rate in the destination regions while lowering the average age of its own population. This suggests that inferences made, say about fertility, on the basis of a model that ignores internal migration may be seriously in error. For example, Table 7A illustrates the significant impact on the ultimate stable age composition and regional share of Region 2 that is occasioned by a doubling and tripling of fertility levels in Region 1 while everything else is held constant. The mean age of the population in Region 2 declines by 5.1 and 8.9 years, respectively, while its regional share decreases by 24 percent in the first instance and by 36 percent in the second.

2.2 Spatial Zero Population Growth

Spatial zero growth, like temporal zero growth, may be viewed either as a condition that ultimately prevails uniformly over space and time, or one that exists only because of a fortuitous balancing of regional rates of positive, negative, and zero growth. Because no obvious advantages arise from the latter case, it is quite natural to view the attainment of temporal zero growth in the long-run in terms of an indefinite continuation of temporal zero growth in the short-run and of spatial zero growth in the long-run in terms of zero growth within each region of a closed multiregional population system. Such a view allows us to confine our attention to the evolution of a particular subset of stationary populations, called spatial zero growth populations, i.e., stable multiregional populations that have a zero growth rate. To derive such populations, we augment the usual twin assumptions of a fixed mortality schedule and a fixed fertility schedule, set at replacement level, with the assumption of a fixed migration schedule. Multiregional populations subjected to such regional growth regimes ultimately assume a persisting zero rate of growth in every region and exhibit zero growth both over time and over space.

Classical stable population theory informs us that a stationary, say female, population arises out of the combination of a fixed survival function, a fixed maternity function, a product-sum of these two functions (the net reproduction rate) that is equal to unity, and an absence of migration.

Let us relax the last of the four conditions by imagining a multiregional population whose long-run evolution follows the multiregional Lotka equation set out earlier in (6). Substituting $\{\underset{\sim}{B}(t)\} = \{\hat{\underset{\sim}{Q}}\}$ gives

$$\{\hat{\underset{\sim}{Q}}\} = [\int_0^\infty \hat{\underset{\sim}{m}}(a)\underset{\sim}{\ell}(a)da]\{\hat{\underset{\sim}{Q}}\} = \hat{\underset{\sim}{R}}(0)\{\hat{\underset{\sim}{Q}}\} \quad , \tag{8}$$

where carets denote stationary population measures.

Equation (8) shows that for a spatial zero growth population to be maintained, the dominant characteristic root of the net reproduction matrix $\hat{\underset{\sim}{R}}(0)$ must be unity. Consequently a reduction of fertility to replacement level may be interpreted as a reduction of the elements of $\underset{\sim}{m}(a)$ to a level that reduces the dominant characteristic root of a given net reproduction matrix $\underset{\sim}{R}(0)$ to unity. Such an operation transforms $\underset{\sim}{m}(a)$ to $\hat{\underset{\sim}{m}}(a)$ and $\underset{\sim}{R}(0)$ to $\hat{\underset{\sim}{R}}(0)$.

The vector $\{\hat{\underset{\sim}{Q}}\}$ in (8) is the characteristic vector associated with the unit dominant characteristic root of $\hat{\underset{\sim}{R}}(0)$ and denotes the total number of births in each region of a spatial zero growth population. The proportional allocation of total births that it defines is directly dependent on the transformation that is applied to change $\underset{\sim}{R}(0)$ to $\hat{\underset{\sim}{R}}(0)$. Since in a spatial zero growth population the regional <u>stationary</u> equivalent population $\hat{Y}_i$ is equal to the quotient formed by $\hat{Q}_i$ and the birth rate $\hat{b}_i$, we see that the different ways in which $\underset{\sim}{R}(0)$ is transformed into $\hat{\underset{\sim}{R}}(0)$ become, in fact, alternative "spatial paths" to a stationary multiregional population.

The dominant characteristic root of the net reproduction matrix of a growing multiregional population is greater than unity, i.e., $\lambda_1[\underset{\sim}{R}(0)] > 1$. If the fertility of each regional <u>cohort</u> of women in this multiregional population were immediately set to replacement level by the multiplication of each <u>region's</u> age-specific birth rates by a fixed fertility adjustment factor, γ_i say, then

$$\{\underset{\sim}{\gamma}\} = [\underset{\sim}{R}(0)']^{-1}\{\underset{\sim}{1}\} \quad .$$

Setting the fertility of each female cohort in every region to bare replacement level, <u>the cohort replacement alternative</u>, is but one of many possible spatial patterns of fertility reduction. One could instead, for example, consider a fertility reduction scheme in which the aggregate system-wide net reproduction rate is reduced to unity through the multiplication of <u>all</u> age-specific birth rates by the <u>same</u> fertility adjustment factor, γ say. In that case

$$\gamma = \frac{1}{\lambda_1[\underset{\sim}{R}(0)]} \quad .$$

This particular spatial pattern of fertility reduction may be called the <u>proportional reduction alternative</u>, and its redistributional impacts can be quite different from those of the cohort replacement alternative.

Mathematical analyses of spatial zero population growth can be facilitated by the adoption of a spatial formulation of R.A. Fisher's (1929) concept of reproductive value. Generalizing the notation of Keyfitz (1975), the reproductive value at age x may be expressed in matrix form as

$$\{\underset{\sim}{v}(x)\}' = \{\underset{\sim}{v}(0)\}' \int_x^\infty e^{-r(a-x)} \underset{\sim}{m}(a) \underset{\sim}{\ell}(a) \underset{\sim}{\ell}(x)^{-1} da \quad , \quad (9)$$

$$= \{\underset{\sim}{v}(0)\}' \underset{\sim}{n}(x) \quad , \text{ say,}$$

where

$$\{\underset{\sim}{v}(0)\}' = \{\underset{\sim}{v}(0)\}' \int_0^\infty e^{-ra} \underset{\sim}{m}(a) \underset{\sim}{\ell}(a) da = \{\underset{\sim}{v}(0)\}' \underset{\sim}{\Psi}(r) \quad .$$

The matrix $\underset{\sim}{n}(x)$ represents the expected discounted number of female offspring per woman at age x. The element $n_{ij}(x)$ gives the discounted number of female children to be born in region j to a woman now x years of age and a resident of region i. The vector $\{\underset{\sim}{v}(x)\}$ represents the reproductive values of x-year old women, differentiated by region of residence. Observe that the elements of $\{\underset{\sim}{v}(x)\}$ depend on the scaling given to $\{\underset{\sim}{v}(0)\}$, the <u>left</u> characteristic vector associated with the unit dominant characteristic root of the characteristic matrix $\underset{\sim}{\Psi}(r)$. Thus in the multiregional model, the reproductive value of a baby girl depends on where she is born.

Equation (9) may be given the following demographic interpretation. If lives are loaned to regions according to the (column) vector $\{\underset{\sim}{Q}\}$ then the amount of "debt" outstanding x years later is given by the (row) vector $\{\underset{\sim}{v}(x)\}'$, the regional expected values of subsequent offspring, discounted back to age x. The elements of this vector, therefore, may be interpreted as spatial (regional) reproductive values at age x.

Spatial reproductive values at age x, $v_i(x)$, may be appropriately consolidated to yield total spatial reproductive values, v_i, by means of the relationship

$$\{\underset{\sim}{v}\}' = \{\underset{\sim}{v}(0)\}'\underset{\sim}{n} \quad ,$$

where $\underset{\sim}{n}$ is a matrix of total discounted number of female offspring associated with that population. The total reproductive value of the multiregional population then is

$$v = \{\underset{\sim}{v}\}'\{\underset{\sim}{1}\} \quad .$$

A numerical illustration mav be instructive at this point. Table 8 shows that under the 1961 regime of fertility, mortality, and migration, the total discounted number of daughters to be born to Yugoslavia's 1961 female population is 5,528,742.* Of these, 383,133 or 6.93 percent will be born in Slovenia, and 379,208 or 6.86 percent will be children of the observed 1961 female residents of Slovenia. Of the ultimate discounted 383,133 female births in Slovenia, 30,404 can be attributed to women now residing in the rest of Yugoslavia and 352,729 to potential mothers now living in Slovenia.

Table 8. Total discounted number of daughters to observed female population by region of birth and region of residence: Yugoslavia, 1961.

Source: Rogers and Willekens (1978)

Region of Birth of Daughter	Region of Residence of Mother		
	Slovenia	Rest of Yugoslavia	Total
Slovenia	352,729	30,404	383,133
Rest of Yugoslavia	26,479	5,119,130	5,145,609
Total	379,208	5,149,534	5,528,742

*The slight discrepancy between this total and the one reported on p.114 of Rogers (1975a) may be attributed to differences in computer hardware.

To derive the total reproductive value of the observed female population one must weight the discounted number of offspring according to region of birth. If we assign a value of unity to a birth in Slovenia then 1.798369 is the corresponding value of a birth in the rest of Yugoslavia. The total reproductive value of Slovenian women is

$$352{,}729(1) + 26{,}479(1.798369) = 400{,}347 \quad ,$$

and the corresponding value for women residing in the rest of Yugoslavia is

$$30{,}404(1) + 5{,}119{,}130(1.798369) = 9{,}236{,}491 \quad .$$

Adding the two subtotals together gives the aggregate system-wide total reproductive value

$$V = 400{,}347 + 9{,}236{,}491 = 9{,}636{,}838 \quad ,$$

for the case where $v_1(0)$ is set equal to unity.

Keyfitz (1975) has shown that if fertility were to drop immediately to replacement level in a population that is closed to migration, the ultimate stationary number of births in the resulting zero growth population would be

$$\hat{Q} = \frac{\hat{V}}{\mu} \tag{10}$$

where μ is the mean age of childbearing in the stationary population, and $\hat{V}$ is the total reproductive value corresponding to an intrinsic rate of growth $r = 0$. The corresponding ultimate stationary total population may be found by dividing $\hat{Q}$ by the stationary birth rate $\hat{b}$ or, equivalently, by multiplying it by $e(0)$, the expectation of life at birth,

$$\hat{Y} = \frac{\hat{Q}}{\hat{b}} = e(0)\hat{Q} \quad . \tag{11}$$

Such a calculation gives the same result as a full population projection carried out with the modified fertility schedule $\hat{m}(a)$.

The above results have their spatial (multiregional) counterparts. To develop these we define $\{\hat{\underset{\sim}{v}}(x)\}'$ to be the vector of spatial reproductive values corresponding to an intrinsic rate of growth r = 0. (We have seen earlier that a transition to zero growth may be carried out by multiplying the fertility schedule $\underset{\sim}{m}(a)$ by the fertility adjustment matrix $\underset{\sim}{\gamma}$.) Then the ultimate number of stationary equivalent births can be shown to be

$$\{\hat{\underset{\sim}{Q}}\} = \frac{1}{\{\hat{\underset{\sim}{v}}_1(0)\}'\underset{\sim}{\mu}\{\hat{\underset{\sim}{Q}}_1\}} \hat{v}\{\hat{Q}_1\} = \frac{\hat{v}}{\mu}\{\hat{\underset{\sim}{Q}}_1\} , \tag{12}$$

where $\{\hat{\underset{\sim}{v}}_1(0)\}'$ and $\{\hat{\underset{\sim}{Q}}_1\}$ are, respectively, the left and right characteristic vectors associated with the unit dominant characteristic root of $\underset{\sim}{\gamma}\underset{\sim}{R}(0)$, and where $\underset{\sim}{\mu}$ is the matrix of mean ages of childbearing in the stationary population that evolves after the decline of fertility to replacement level.

The ultimate total stationary population is

$$\{\hat{\underset{\sim}{Y}}\} = \hat{\underset{\sim}{b}}^{-1}\{\hat{\underset{\sim}{Q}}\} = \underset{\sim}{e}(0)\{\hat{\underset{\sim}{Q}}\} , \tag{13}$$

where

$$\hat{\underset{\sim}{b}} = [\int_0^\infty \hat{\underset{\sim}{m}}(a)\underset{\sim}{\ell}(a)da][\int_0^\infty \underset{\sim}{\ell}(a)da]^{-1} = \underset{\sim}{\gamma}\underset{\sim}{R}(0)\underset{\sim}{e}^{-1}(0) ,$$

and $\underset{\sim}{e}(0)$ is a matrix of expectations of life at birth disaggregated by regions of birth and residence.

Equation (12) has a simple and intuitively appealing interpretation. Consistent with (10) it defines the total size of stationary equivalent births in a multiregional population to be equal to the quotient of the total reproductive value $\hat{v}$ and the weighted index $\mu = \{\hat{v}_1(0)\}'\mu\{\hat{Q}_1\}$ in that population, both evaluated after the decline in fertility to replacement level, and distributes that total according to the proportional allocation determined by the right characteristic vector associated with the unit dominant characteristic root of the modified net reproduction rate matrix $\gamma R(0)$. The interpretation of equation (13) follows in a straightforward manner.

We have seen that the geographical distribution of a spatial zero growth population depends very fundamentally on three matrices: $e(0)$, $R(0)$, and γ. The first describes the multiregional levels of mortality and migration; the second sets out the multiregional net reproduction patterns before the decline in fertility; and the third defines the particular "spatial path" by which fertility is reduced. The product $\gamma R(0)$ gives $\hat{R}(0)$, whose characteristic vector associated with the unit root and scaled to sum to $\hat{Q}$ is $\{\hat{Q}\}$.

This dependence suggests a crude but effective procedure for estimating the <u>momentum</u> of spatial zero population growth. One begins by first estimating the ultimate size of total stationary equivalent births $\hat{Q}$, by means of Keyfitz's (1975) <u>momentum</u> formula; that total then may be distributed among the various regions according to the allocation defined by the characteristic vector associated with the unit root of $\gamma R(0)$; and, finally, the resulting vector may be premultiplied by $e(0)$ to find $\{\hat{Y}\}$.

2.3 Intervention

Public concern over population matters generally arises when the demographic acts of individuals affect societal welfare to produce a sharp divergence between the aggregation of individual net benefits and social well-being. In such situations, population processes properly become the focus of public debate and the object of public policy.

Because a policy to increase mortality is not only politically infeasible but also morally offensive, reductions in the size of regional populations must be brought about by reductions in their birth rates or by some control of internal migration.

The effects of birth or migration control in a multiregional population system governed by the growth dynamics defined in equation (5) may be introduced by an intervention vector, $\{\underset{\sim}{f}\}$ say, which is added to the population in each time period (Rogers, 1968, p.53):

$$\{\underset{\sim}{K}(t + 1)\} = \underset{\sim}{G}\{\underset{\sim}{K}(t)\} + \{\underset{\sim}{f}\} \quad . \tag{14}$$

Starting with an initial population distribution at a given moment in time t = 0, we may trace out the cumulative impact of a particular intervention vector, acting under an unchanging growth regime, by repeatedly applying (14) to derive (Rogers, 1971, p.99):

$$\{\underset{\sim}{K}(t)\} = \underset{\sim}{G}^{t}\{\underset{\sim}{K}(0)\} + (\underset{\sim}{I} - \underset{\sim}{G})^{-1}(\underset{\sim}{I} - \underset{\sim}{G}^{t})\{\underset{\sim}{f}\} \quad .$$

Assuming now that a vector of target populations at the planning horizon year T, has been defined, the intervention vector that will bring this about is readily calculated as:

$$\{\underset{\sim}{f}\} = (\underset{\sim}{I} - \underset{\sim}{G}^{T})^{-1}\left[(\underset{\sim}{I} - \underset{\sim}{G})[\{\underset{\sim}{K}(T)\} - \underset{\sim}{G}^{T}\{\underset{\sim}{K}(0)\}]\right] \quad .$$

Paul Drewe (1971) has used the above intervention model to demonstrate that a rather major redirection of internal migrants would be necessary to achieve national plans for regional population targets in the year 2000 for the three northern provinces of the Netherlands (Groningen, Friesland, and Drenthe). In a more recent paper, he updates his analysis in the light of more current data and a revised plan (Drewe, 1977).

Frans Willekens (1976) has developed the intervention perspective much further in his dissertation. He shows that the model in (14) may be usefully extended along three important directions:

1) the introduction of economic control variables and the specification of their impacts on the population distribution;

2) the expansion of the initial period control problem to a truly dynamic control problem; and

3) the admission of other constraints on both the state and the control variables, and the formulation of policy objectives in terms of variables other than population targets.

A fundamental feature of population policy is the non-demographic character of its goals and instruments. Control of migration flows is rarely justified solely on the grounds of achieving target population totals. Nor is the control exercised directly on population flows. Rather, the goals and interventions are expressed in terms of economic variables such as regional incomes, employment, housing construction, and government expenditures. Therefore, let $\{\underset{\sim}{u}\}$ be a vector of socioeconomic control variables and, for the sake of simplicity, assume the linear relationship $\{\underset{\sim}{f}\} = \underset{\sim}{A}\{u\}$, where $\underset{\sim}{A}$ is a time invariant coefficient matrix. An element a_{ij} denotes the impact of the j^{th} control variable on the i^{th} element of $\{\underset{\sim}{f}\}$. Substituting this relationship into (14) gives

$$\{\underset{\sim}{K}(t + 1)\} = \underset{\sim}{G}\{\underset{\sim}{K}(t)\} + \underset{\sim}{A}\{u\} \quad . \tag{15}$$

Equation (15) links the population distribution at a given time to the population distribution at a preceding point in time, and to socioeconomic policy variables. The model is closely related to the static policy model developed by Tinbergen (1963). A solution exists if the rank of $\underset{\sim}{A}$ is equal to the number of targets. The solution is unique if $\underset{\sim}{A}$ is nonsingular, i.e., if, in the jargon of Tinbergen, the number of instruments is equal to the number of targets. In that case,

$$\{\underset{\sim}{f}\} = \underset{\sim}{A}^{-1}[\{\underset{\sim}{K}(t + 1)\} - \underset{\sim}{G}\{\underset{\sim}{K}(t)\}] \ .$$

The policy models in (14) and (15) are not truly dynamic. Although the control vector varies over time, its trajectory is fixed once the instruments of the initial time period have been chosen. Relaxing this restriction leads to the multiperiod control model

$$\{\underset{\sim}{K}(t + 1)\} = \underset{\sim}{G}\{\underset{\sim}{K}(t)\} + \underset{\sim}{A}\{\underset{\sim}{u}(t)\} \quad , \qquad (16)$$

and its solution,

$$\{\underset{\sim}{K}(t)\} = \underset{\sim}{G}^{t}\{\underset{\sim}{K}(0)\} + \sum_{i=0}^{t-1} \underset{\sim}{G}^{(t-1-i)}\underset{\sim}{A}\{\underset{\sim}{u}(i)\} \quad .$$

Two multiperiod policy problems now may be studied:

1) the <u>horizon-oriented policy problem</u>, in which one seeks a sequence of control vectors $\{\underset{\sim}{u}(i)\}$ that guide the evolution of the initial population distribution $\{\underset{\sim}{K}(0)\}$ toward a target vector at time T, assuming fixed coefficient matrices; and

2) the <u>trajectory-oriented policy problem</u>, in which the principal question addressed is whether there exists a sequence of control vectors $\{\underset{\sim}{u}(i)\}$ such that any sequence of target vectors can be realized, given a specific initial condition and unchanging coefficient matrices.

In mathematical systems theory, the first policy problem is known as state controllability. The second problem is called complete state controllability. Both are formally analyzed in Willekens (1976).

The policy models considered thus far assume that the policymaker's objectives can be expressed completely in terms of population targets, and that the achievement of these targets is constrained only by the equation that describes the system's dynamic behavior. No direct constraints were placed on population totals, and the control variables were constrained only through the introduction of linear dependencies.

In practical policy applications, the values taken on by population and control vectors are likely to be restricted by political and socioeconomic considerations. This suggests the desirability of adding instrumental variables to population variables to define an explicit objective function.

It also may be desirable to constrain each element of the control vector inside of a lower and upper bound:

$$\underline{u}_i(t) \leq u_i(t) \leq \bar{u}_i(t) \quad ,$$

and to assume a budget constraint for each period:

$$\{\underset{\sim}{c}(t)\}'\{\underset{\sim}{u}(t)\} \leq C(t) \quad ,$$

and for the entire span of control:

$$\sum_{t=0}^{T-1} \{\underset{\sim}{c}(t)\}'\{\underset{\sim}{u}(t)\} \leq C \quad .$$

The cost vector $\{\underset{\sim}{c}(t)\}'$ contains the unit costs incurred by the use of each instrument.

The above constraints refer to the control vector. It also may be desirable to incorporate constraints on the population distribution vector itself. For example, the policy-maker may wish to define lower and upper bounds for the size of the population in each region in order to avoid social costs arising out of excessive density or of excessive depopulation. If the constraints set and the objective function are both linear, the policy model may be expressed as a dynamic linear programming problem (Propoi and Willekens, 1978). If the objective function is quadratic, the computational task is considerably more complex (Evtushenko and MacKinnon, 1976).

The most general formulation of a dynamic population policy problem may be conveniently expressed as an <u>optimal control problem</u> with (i) a state equation describing the dynamics of the system, (ii) a set of constraints on the state and control variables, (iii) a set of boundary conditions, and (iv) an objective function

Such a formulation combines several fundamental themes in two related but largely independent bodies of literature: the mostly mathematical literature in systems engineering that deals with the control of complex systems describable by sets of differential or difference equations, and the more substantive literature in the formal theory of economic growth and policy. The logical structures of the two paradigms are similar, and their formalisms can be fruitfully transferred to the field of population policy (Willekens and Rogers, 1977).

2.4 Urbanization

Urbanization is a finite process all nations go through in their transition from an agrarian to an industrial society. Such urbanization transitions can be depicted by attenuated S-shaped curves which tend to show a swift rise around 20 percent, a flattening out at a point somewhere between 40 and 60 percent, and a halt or even a decline in the proportion urban at levels above 75 percent.

A large proportion of the population of the less developed world is engaged in agriculture. In consequence, a relatively small fraction of this population is urban: only about one-fourth. The corresponding fraction for the developed world is close to seven-tenths. But because of their much larger share of the world's population, less developed countries today have as large an urban population as do the developed countries: just under four-fifths of a billion people each.

Accelerated rates of population growth and urbanization are direct consequences of higher rates of natural increase (births minus deaths) and of net urban migration (urban inmigration minus urban outmigration). Explanations of temporal and spatial variations in the patterns exhibited by these two sets of rates generally have taken the form of descriptive generalizations phrased in terms of "transitions" or "revolutions". Specifically, the vital revolution is commonly held to be the process whereby societies with high birth and death rates move to low birth and death rates. The mobility revolution is the transformation experienced by societies with low migration rates as they advance to a condition of high migration rates. These two revolutions occur simultaneously and they jointly constitute the demographic transition.

Urbanization results from a particular spatial interaction of the vital and the mobility revolutions. It is characterized by distinct urban-rural differentials in fertility-mortality levels and patterns of decline, and by a massive net transfer of population from rural to urban areas through internal migration.

In a now classic analysis of the demoeconomic consequences of fertility reduction, Ansley Coale (1969) examined some of the ways in which the population characteristics of less developed countries are related to their poverty and how alternative demographic trends might affect their modernization.

> We shall be concerned here with the implications, for the growth in per capita income and for the provision of productive employment, of alternative possible future courses of fertility. The specific alternatives to be considered are the maintenance of fertility at its current level and, as the contrasting alternative, a rapid reduction in fertility, amounting to fifty percent of the initial level and occupying a transitional period of about twenty-five years. (Coale, 1969, p.63)

After generating the two alternative projections or "scenarios", Coale went on to

> inquire what effects these contrasting trends in fertility would have on three important population characteristics: first, the burden of dependency, defined as the total number of persons in the population divided by the number in the labor force ages (fifteen to sixty-four); second, the rate of growth of the labor force, or, more precisely, the annual per cent rate of increase of the population fifteen to sixty-four; and third, the density of the population, or, more precisely, the number of persons at labor force age relative to land area and other resources. (Coale, 1969, p.63)

In a recent paper (Rogers, 1977) we adopted Coale's scenario-building approach to focus on some of the demoeconomic consequences of rapid urbanization. We began by developing four alternative population scenarios and then went on to examine the implications that these alternative trends in migration and fertility would have on Coale's three important population characteristics: the dependency burden, the growth rate of labor force "eligibles", and the density of the population.

As in the Coale paper, a hypothetical initial population of one million persons with an age composition and fertility-mortality rates typical of a Latin American country was projected one hundred and fifty years into the future. To his alternative projections (A, fertility unchanged and B, fertility reduced), however, we added two others by varying our assumptions on internal migration (a, migration unchanged and b, migration increased). This produced the following four possible combinations:

	a. Migration unchanged	b. Migration increased
A. Fertility unchanged	Projection Aa	Projection Ab
B. Fertility reduced	Projection Ba	Projection Bb

Coale's assumptions on initial and future patterns of mortality and fertility were a crude birth rate of about 44 per 1,000 and a crude death rate of 14 per 1,000, giving rise to a population growing at 3 percent per year. Starting with an expectation of life at birth of approximately 53 years, he assumed that during the next 30 years it will rise to about 70 years, at which point no further improvement will occur. In Coale's Projection A current age-specific rates of childbearing are fixed for 150 years; in Projection B they are reduced by 2 percent each year for 25 years, (reducing fertility to half of its initial level), at which point they too are fixed for the remainder of the projection period.

For our four urbanization scenarios we spatially disaggregated Coale's data and assumptions in the following manner. Twenty percent of the initial population of a million persons was taken to be urban. The initial values for birth and death rates were assumed to be lower in urban areas than in rural areas (40 against 45 per thousand for the birth rate, and 11 against 15 per thousand for the death rate). Mortality and fertility were reduced as in the Coale projections, but the declines were assumed to occur ten years sooner in urban areas (25 instead of 35 years for the decline in mortality, and 20 instead of 30 years for the decline in fertility).

A multiregional population projection also requires a specification of the initial values and future course of internal migration. To generate the four scenarios, initial rates of outmigration were set equal to those prevailing in India in 1960 (Bose, 1973); that is, a crude outmigration rate from urban areas of 10 per 1000 and a corresponding rate from rural areas of 7 per thousand. The age-specific rates of outmigration from urban areas were held fixed in all four projections, as were the corresponding rates from rural areas in the two "a" projections. Outmigration from rural areas in the two "b" projections, however, was assumed to increase six-fold over a period of 50 years and then to drop to half its peak value over the following 30 years, after which it was held unchanged for the remaining 70 years of the projection period.

The assumptions appear to be reasonable in that the hypothetical urbanization paths they chart are plausible. For example, the percentage-urban paths for the "b" projections resemble the general shape of historically observed urbanization paths, and the trajectories of urban and rural growth rates for these projections are in general similar to those exhibited by data for several developed nations.

As in Coale's scenarios, the initial population and the future regime of mortality are the same for all of the four population projections summarized in Figure 6. The major impact of the drop in fertility appears in the projected totals: the "A" projection totals are about 24 times as large as the "B" projection totals after 150 years. Migration's impact, on the other hand, appears principally in the spatial distribution of these

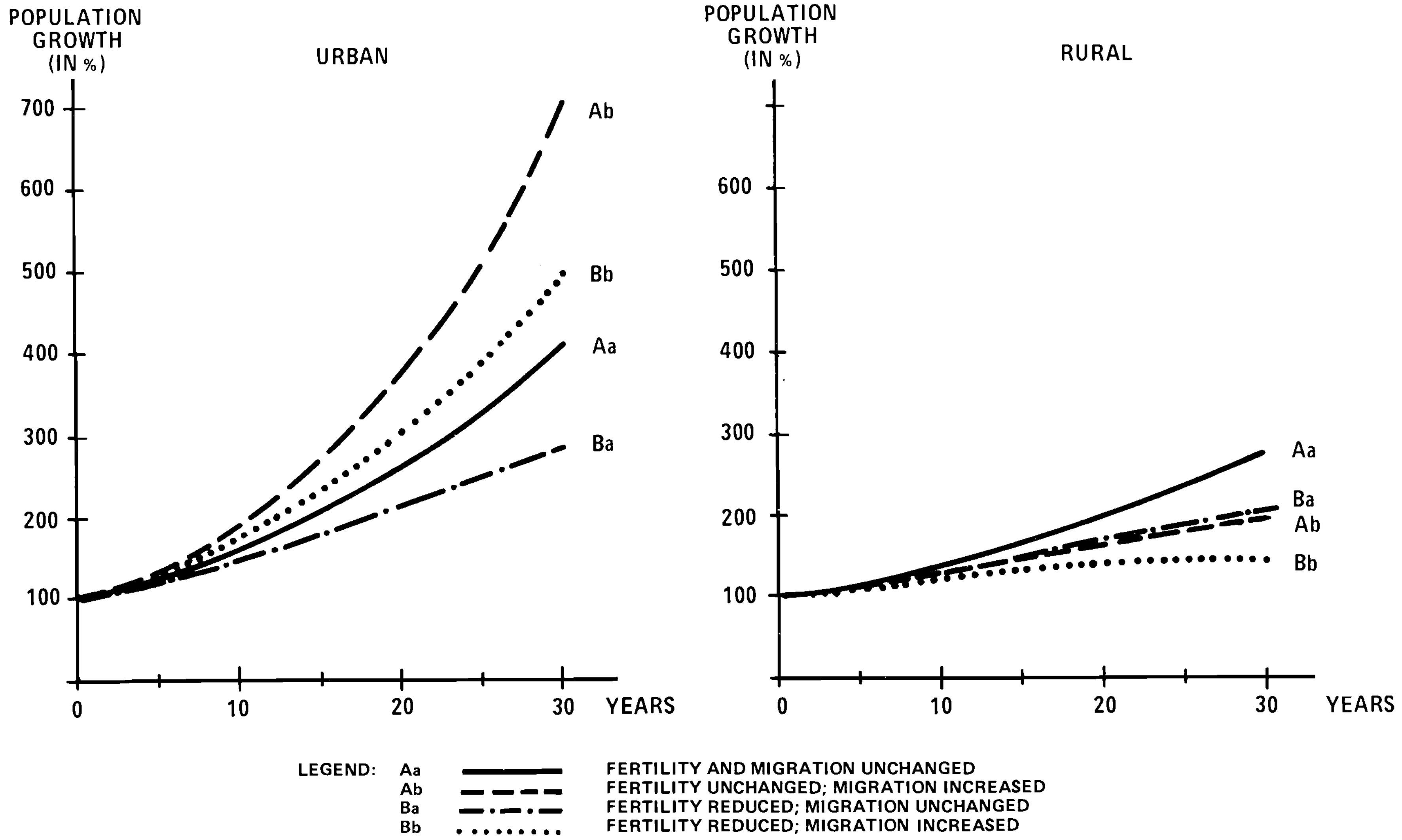

Figure 6. Alternative projections of the population of a less developed country: four scenarios.
Source.: Adapted from Rogers (1977), pp.29-30.

totals: the "a" projections allocate approximately a third of the national population to urban areas after 150 years, whereas the "b" projections double this share.

The principal demographic impacts of reduced fertility described by Coale are not altered substantially by the introduction of migration as a component of change and the concomitant spatial subdivision of the national population into urban and rural sectors. Figures 7 and 8 show that for a given regime of migration (a or b), the major impacts of reduced fertility are, as in the Coale model: a decline in the burden of dependency in the short run, a lowering of the growth rate of the labor force population in the medium run, and a very much lower density of people to resources in the long run. The spatial model, however, does bring into sharp focus urban-rural differentials: (1) in dependency burdens and in the relative magnitudes of their decline following fertility reduction, and (2) in initial growth rates of the labor force population and the paths of their gradual convergence in the long run.

The dependency ratio in urban areas is 19 points lower than its rural counterpart at the start of the projection period. With constant fertility, the regional dependency burdens remain essentially unchanged. Declining fertility, however, narrows these differentials to almost a third of their original values, as the urban drop of 33 points is matched by a corresponding decline of 45 points in rural areas.

The annual growth rates of the labor force population in urban and rural areas initially are 0.05 and 0.03, respectively. For both migration regimes, however, they converge to approximately the same values in the long run: 0.04 in the constant fertility scenario and slightly above 0.01 in the reduced fertility projection.

The major demographic impacts of increased rural-urban migration for a given regime of fertility, as set out in Figure 7 and 8, are negligible with respect to dependency burdens and are of paramount importance, in the short and medium runs, with regard to the growth rate of the population aged 15 - 64. In the long run migration also has a moderately powerful impact on the density of workers to resources in rural areas.

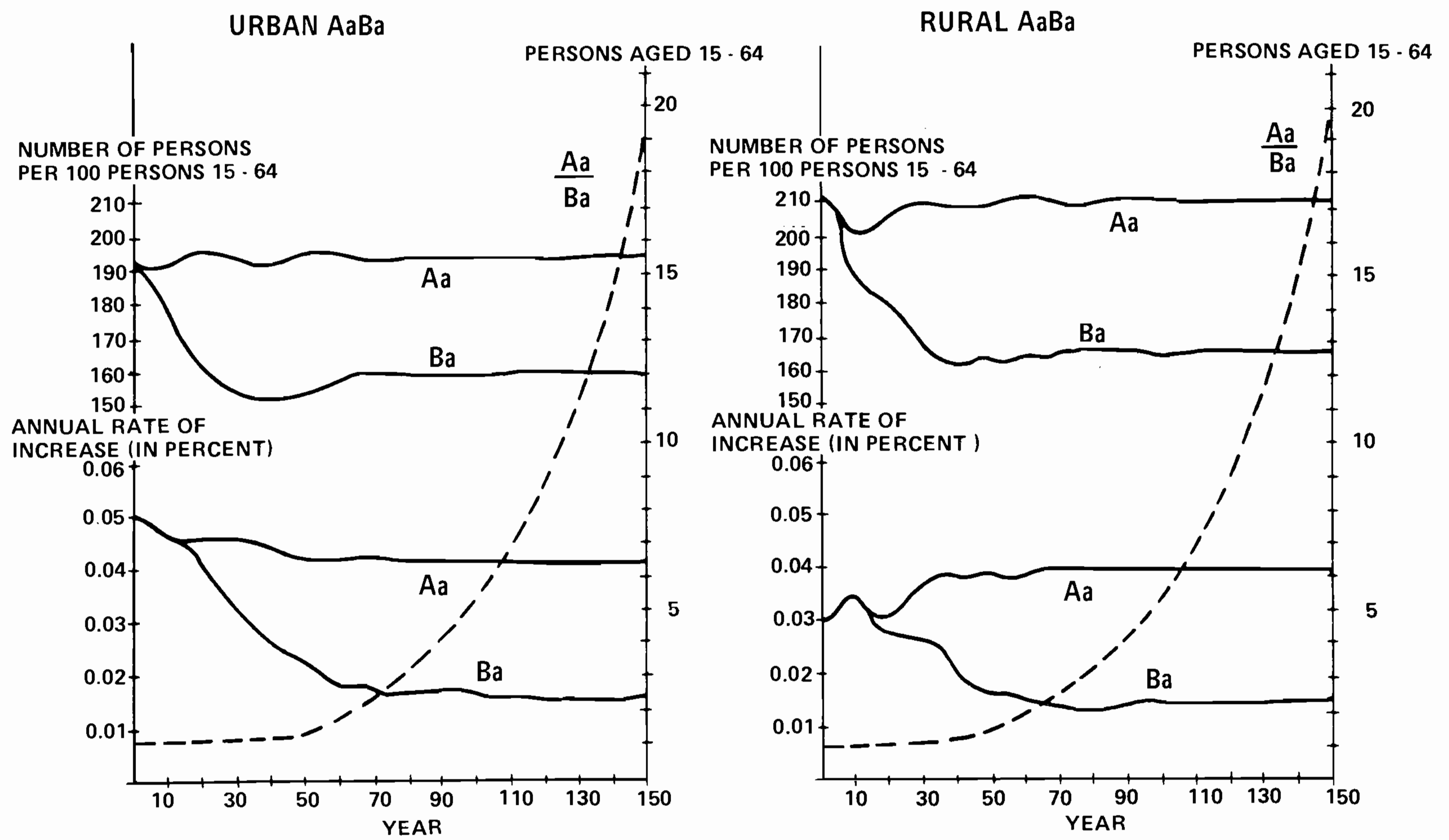

Figure 7. Dependency burden, annual rate of increase, and relative size of population aged 15 - 64 years: alternative urban-rural projections, migration unchanged.

Source: Rogers (1977), p. 40.

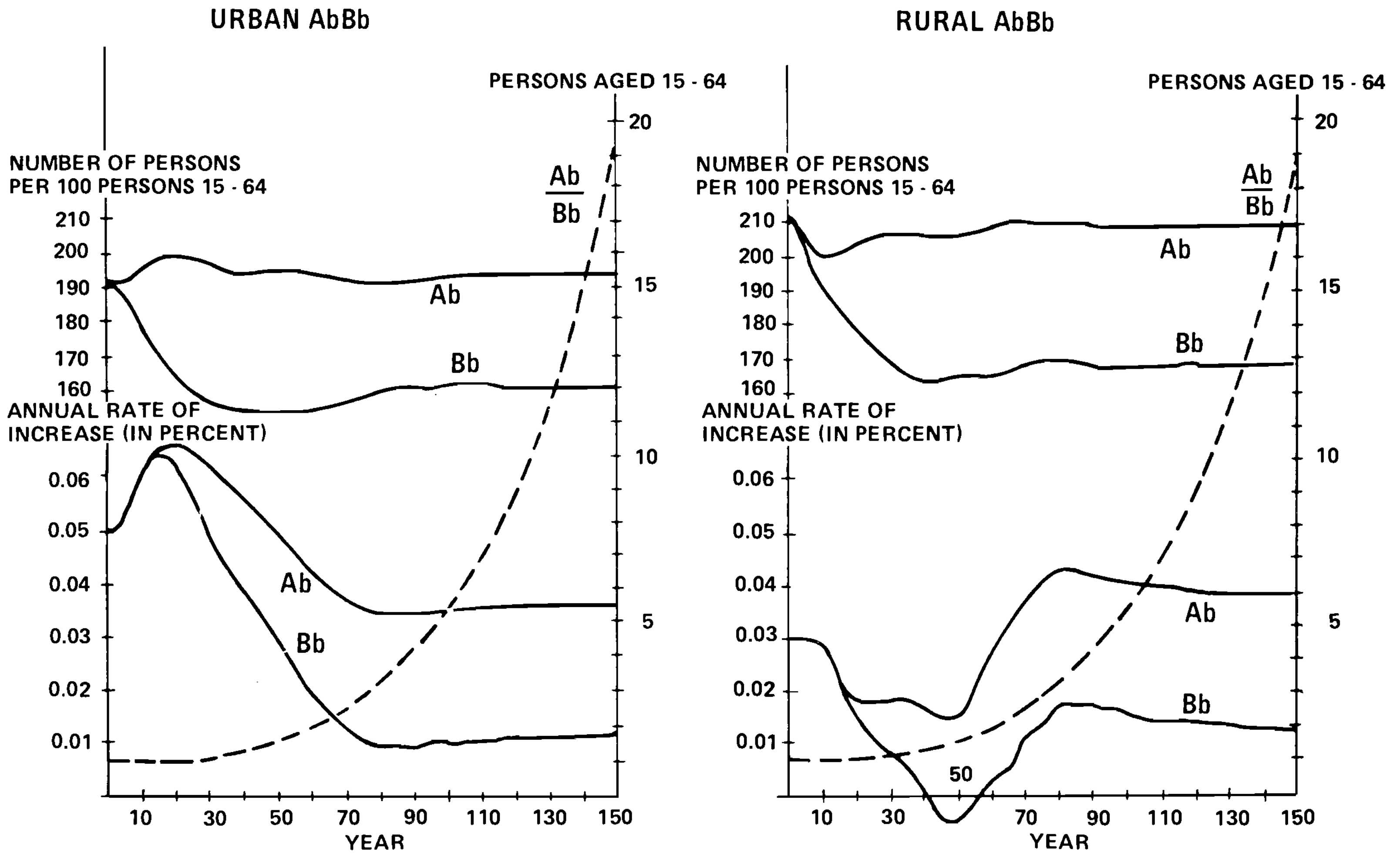

Figure 8. Dependency burden, annual rate of increase, and relative size of population aged 15 - 64 years: alternative urban-rural projections, migration increased.

Source: Rogers (1977), p. 41.

Perhaps the most interesting observation suggested by the scenarios is the transitory nature of high rates of urban growth. In the "b" projections, urban growth rates in excess of 6 percent per annum occur only in the short run, as the national population is in its early phases of urbanization. This sudden spurt of growth of urban areas in the short run declines over the medium run, and in the long run levels off at a rate below that generated by the fixed migration regime. The growth curve of rural areas, of course, assumes a reverse trajectory, with the growth of the rural working population declinining to relatively low, even negative, levels before increasing to stabilize at about the same level as that prevailing in the urban population.

It has been said that models are always based on assumptions known to be false, and that this is what differentiates them from the phenomena they purport to describe. Demographic models are no exception to this dictum, and all population projections, for example, are generated on the basis of assumptions that are almost certain to be violated. One cannot foresee the future, and important insights into the dynamics of human populations can indeed be revealed by relatively simple linear models based on rather restrictive assumptions. As has been demonstrated in this paper, such models can be used to structure data collection efforts; they often generate hypotheses for empirical confirmation; they can suggest potential policy problems and issues; and they provide indices for comparative studies (Keyfitz, 1971).

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